Guido Ubaldo Marquis del Monte

Mechanicorum liber

Pesaro 1577

Preface of Guido Ubaldo Marquis del Monte

To Francisco Maria II Illustrious Duke of Urbino

There are two qualities, Illustrious Prince, that are usually very effective in adding to men's power, namely, utility and nobility.It seems to me that these join in making the subject of mechanics attractive and in rendering it desirable in comparison with all others.For if we measure the nobility of something by its origin (as most people now do), the origin of mechanics is, on one side, geometry and, on the other side, physics.From the union of these two comes the most noble of the arts, mechanics.For if we hold that nobility is related both to the underlying subject matter and to the logical necessity of the arguments (as Aristotle on occasion asserts), we shall doubtless consider [mechanics] I the noblest of all.It not only crowns and perfects geometry (as Paptus attests) but also holds control of the realm of nature.For whatever helps manual workers, builders, carriers, farmers, sailors, and many others (in [apparent] opposition to the laws of nature)-all this is the province of mechanics.And mechanics, since it operates against nature or rather in rivalry with the laws of nature, surely deserves our highest admiration.

Now it is certainly true, and freely admitted by anyone who learned it previously from Aristotle, that all mechanical problems and all mechanical theorems are reducible to the wheel and depend therefore on its principle, which is apprehended no less by the senses than by reason.The wheel is the device that is best adapted to movement, and the more so the larger it is.

In addition to this nobility, [mechanics possesses] the highest utility in matters pertaining to life.And this utility exceeds all other forms of utility obtained from the various arts for the reason that other branches unfolded their usefulness after a long interval of time had elapsed from the birth of the world; but mechanical usefulness was so necessary to men even from the very beginning of the world that, had it been removed, the sun itself would have been removed from the world. For under whatever constraints the life of Adam was passed, though he may have had to ward off the onslaughts from the sky in huts thatched with straw and in narrow hovels and cottages, and by clothing his body, and though he himself may have been concerned only with warding off the rain, snow, wind, sun, and cold-whatever was the case, the situation was all mechanical.

And the case with this subject [mechanics] is not the same [historically] as that of winds which are strongest in the place where they arise, but arrive at a distant point broken and weakened.But rather the [historical] situation is the same as generally occurs with great rivers, which, while they are small where they rise, are continually increased and have a broader bed the farther they are from their source.Thus with the passing of time mechanical power began to distribute equally the labor of plowing by the use of the yoke and to have the plow drawn around the field.

And then mechanical power showed men how, with teams of two and four animals, to move produce, merchandise, and all kinds of cargo- to export from our country to neighboring lands and, again, to import from those lands to our own. Moreover, when things came to be measured not merely by necessity but by their beauty and usefulness, it was thanks to mechanical ingenuity that we could move ships by oars; that with a small rudder at the extremity of the stern we could steer huge triremes; that often with the hands of one individual, rather than with the hands of many workmen, we could now lift heavy stones and beams for our construction workers and architects; and now with a kind of swing-beam we could draw water from wells for gardeners.

By mechanical power, too, are wines, oils, and unguents pressed out by presses used for liquids and forced to give up to the owner whatever liquid they contain.Hence, too, by means of two forces pulling in opposite directions we have divided stout trees and great masses of marble.Hence also in war, in the building of ramparts, in fighting at close quarters, in attacking and defending places, there are almost infinite uses [of mechanics].

With the help of mechanics, too, those who work with wood, stone and marble, wines, oils and unguents, iron, gold, and other metals, as well as surgeons, barbers, bakers, tailors, and all workers in the useful arts, make many important contributions to human life.

And as for certain recent manipulators of words who deprecate mechanics, let them go and wipe away their shame, if they have any, and stop falsely charging [mechanics with] lack of nobility and lack of usefulness.If they still do not wish to do so, let us leave them, I say, in their ignorance; and let us rather follow Aristotle, the leader of the philosophers, whose burning love for mechanics is sufficiently proved by the acute Questions of Mechanics which he gave to posterity.In this achievement he greatly surpassed Plato.For, when Archytas and Eudoxus were keenly exploring the usefulness of mechanics, Plato (as Plutarch tells us) discouraged them from this course, on the ground that they were revealing to the masses and making public the noblest possession of philosophers and betraying, as it were, the secret mysteries of philosophy.But surely, at least in my judgment, such a view [that is, Plato's] is to be completely rejected, unless perhaps we wish to praise the detached contemplation of so noble a subject yet to impugn the fruits, the usefulness, and the goal of the art.

But in comparison with all other mathematicians Archimedes alone is to be praised most eloquently, for God willed that in mechanics he should be a unique ideal which all students of that subject might keep before them as a model for imitation.For he made a model of the universe all enclosed in a quite small and fragile glass sphere, with stars that imitated the actual work of nature and so accurately exhibited the laws of the heavens by their precise motions that the hand that rivaled nature deserved the following encomium: "So does his hand imitate nature that nature herself is thought to have imitated his hand."Archimedes, with the help of a block and tackle, pulled a load of 5000 pecks with one hand. Alone with his machines he pulled a heavily loaded ship onto the shore and then pulled it toward himself as if it were being moved in the sea by oars or sails.And then he pulled it from the shore back into the sea (something that all the [human] strength of Sicily could not have accomplished).His too are those engines of war with which Syracuse was so defended against Marcellus that the operator of those engines was always called a hundred-handed Briareus by the Romans.Finally, relying on this art he made so bold as to give utterance to a statement in such [apparent] conflict with the laws of nature: "Give me a place to stand, and I shall move the earth. "And not only do we show in the present book that this could have been done with a lever, but, in fact, all antiquity seems to me to have been completely convinced of this (though possibly this will appear remarkable to many).For antiquity attributed to Neptune a trident, like a lever; and by virtue of it he is everywhere called Earth-shaker by the poets.Indeed, it is with this in mind that our most celebrated poet introduces Neptune raising the shoals with that device so that they may be visible to the Trojans, "with his trident he raises and opens up the vast shoals".

Other mechanicians were Hero, Ctesibius, and Pappus.And though they did not perhaps reach the pinnacle of mechanics, as did Archimedes, still they had remarkable understanding of the subject of mechanics and were all great men.Indeed this is especially true of Pappus, so that no one could, I believe, blame me for following him as my leader.I have more readily done so for the reason that Pappus does not depart even a nail's breadth from the principles of Archimedes.For I have always wished in this branch of science to follow in the footsteps of Archimedes.And though his thoughts on the subject of mechanics have for some years been widely sought by scholars, still his very learned book On [Plane] Equilibrium is extant; in that book I believe that practically all the teachings of mechanics are gathered together, as in an abundant store. Surely, if the mathematicians of our time had a better knowledge of this book, they would have found that many ideas, which they themselves now declare valid and correct, are there very acutely and properly shaken and overturned.But let them see for themselves.I return to Pappus, who, deeply devoted to a richer application of mathematics and to increasing the profits to be derived from such application, made a thorough and brilliant investigation of the five primary machines, that is, the lever, pulley, wheel and axle, wedge, and screw.And he proved that, in the case of machines, everything that could properly be considered as sharply defined or definitely established was reducible to those machines which [potentially] are capable of unlimited force.I wish that the ravages of time had not caused any loss in the writings of so great a man.For such a thick mist of ignorance would not have covered almost all the earth, nor would there have been such ignorance of the subject of mechanics that men are thought of as leading mathematicians who, by their inept distinctions, remove some difficulties, but not those that are very arduous or obscure.

Thus, there are found some keen mathematicians of our time who assert that mechanics may be considered either mathematically, removed [from physical considerations], or else physically.As if, at any time, mechanics could be considered apart from either geometrical demonstrations or actual motion! Surely when that distinction is made, it seems to me (to deal gently with them) that all they accomplish by putting themselves forth alternately as physicists and as mathematicians is simply that they fall between two stools, as the saying goes.For mechanics can no longer be called mechanics when it is abstracted and separated from machines.

Yet in the midst of that darkness (though there were also some other famous names), Federico Commandino shone like the sun; he, by his many learned studies, not only restored the lost heritage of mathematics, but actually increased and enriched it.For that great man was so endowed with all mathematical talents that Archytas, Eudoxus, Hero, Euclid, Theon, Aristarchus, Diophantus, Theodosius, Ptolemy, Apollonius, Serenus, Pappus, and even Archimedes himself (for his commentaries on Archimedes smell of Archimedes' own lamp) seem to have lived again in him.And, lo, just as he had been suddenly thrust from the darkness and prison of the body (as we believe) into the light and liberty of mathematics, so at the most inopportune time he left mathematics bereft of its fine and noble father and left us so prostrate that we scarcely seem able even by a long discourse to console ourselves for his loss.And yet in his endless concern with the elucidation of other parts of mathematics, he either left mechanics completely untreated or touched on it just casually.

Therefore I began to devote myself more eagerly to this study, and, in making my way through every branch of mathematics, I never lost sight of my course to find whatever could be appropriated and derived from each of these branches, so that I may be better equipped to perfect and embellish mechanics.But now I think that, while I have not completed the treatment of everything that pertains to mechanics, still I have advanced to such a point that I can bring some help to those who learned from Pappus, Vitruvius, and others, what a lever is, a pulley, wheel and axle, wedge, and screw, and how they should be arranged so that weights may be moved, and the many properties present in those machines by virtue of the lever, properties connecting force and weight, which they are eager to learn about.For that reason I thought it was the proper time for me to emerge and give some example of the work I did on this subject.

Now, in order that my whole work might be more easily built up from its foundation to its very top, certain properties of the balance had to be treated, particularly the case when one arm of the balance is depressed by a single weight.On this subject it is strange what disastrous errors were made by Jordanus (who has enjoyed the greatest authority among recent writers) and others who proposed to discuss this subject.Surely it was a difficult task that we undertook, one perhaps beyond our powers.Still, aspiring to notable results, as we do, we deserve to have our efforts and industry meet with the everlasting approval and applause of all good people, since we have devoted all our powers to a study so noble, so magnificent, and so praiseworthy.

This study, such as it is, we have decided to dedicate to you, illustrious Prince, and, surely, there are many obvious reasons for this plan and decision of ours.First there are your hereditary services to our family, by which you have laid us under such an obligation that we readily recognize that we ought to be prepared to give up our blood and even our lives in keeping with your worth.In addition, there is the fact, of no small import, that from boyhood you were so inflamed with a passion not only for all studies, but especially for mathematical studies, that you would consider your life bitter and unhappy unless you had embraced them.And then, occupied in the study [of mathematics], you passed the first part of your life in gaining an understanding of the subject, and often raised your voice, as was worthy of a prince, to say that you were especially fond of mathematics for the reason that mathematics in particular can emerge from that domestic and private kind of life into the sun and dust, as they say.And, indeed, in clear proof of these [public] interests would be the ardent desire for military skill that you manifested from early youth; but it would only reveal my limited mind, were I to try to set forth the things that could be expected from you.For when you were quite a young man you went on to the early accomplishment of many outstanding things.Thus, when the foundations had been laid by his Holiness Pope Pius V for the sound union of Christian princes, you, eagerly essaying to overcome the enemies of Christ, won for yourself true and solid glory.And every time there was deliberation on high policy, you uttered sentiments which showed the highest prudence joined with the greatest elevation of spirit.I shall omit your many other outstanding and heroic acts performed in those times, lest I seem to you to be announcing things which are already known to all.Though all these achievements are great and outstanding, men still await from you achievements far greater and more outstanding.Farewell, then, noblest adornment of the world, and, if at any time you have some leisure, do not disdain to examine these products of my study.

Dedicatory Letter of Filippo Pigafetta
To the Illustrious Signor Giulio Savorgnano Count of Belgade, etc.

My Revered Lord:-Inasmuch as the science of mechanics is highly useful to many and important actions in our lives, there is good reason that philosophers and ancient kings gave it more than a little study and that princes favored excellent engineers and enriched them.Certainly this science is of the highest theoretical value and of subtlest structure, for it deals with that part of philosophy which treats of the elements in general, and of the motion and rest of bodies according to their positions; thus we assign the cause of their natural movements, and thus by machines we force bodies to leave their natural places, carrying them upward and in every direction, contrary to their nature.

Both these goals are carried out by propositions arising from matter itself and artificial structures and instruments.And thus it is necessary to consider this subject in two manners: one that regards theory and the application of reason to things that must be done, making use of arithmetic and geometry, astrology, and natural philosophy; the other that is carried out in practice and requiring activity and manual labor, utilizing architecture, painting, design, the arts of the builder and carpenter and mason and related crafts, and in such a way that these things become intertwined and are a mixture of natural philosophy, mathematics, and the practical arts.So that whoever is instructed by clever men and has learned from childhood the previously mentioned sciences and can also design and work with his hands may become a skilled mechanic, inventor, and maker of marvelous works.

This knowledge includes infinite things of use to men in war and peace; it is useful to cities, farms, and commerce; medicine takes from it the devices for restoring broken and dislocated bones to their places.Thus Oribasius in his book on machines includes many instruments taken from mechanics and converted to medical uses, as the tripaston of Archimedes .The art of navigation receives other aids, such as the rudder which, placed behind or beside the vessel, moves and guides it easily, though it is very small with respect to the whole vessel; oars, which like levers drive it forward; and masts and sails.Windmills, watermills, mills turned by living beings, wagons, plows, and other farm devices are reducible to mechanics.So are the weighing of things with balances, the drawing of water from wells by pulleys or by cranes, called in Latin tollenones, which are like huge balances.The manner of conducting water and raising it from deep valleys to heights is similarly derived.The ancients called those persons mechanics also who produced miraculous effects by means of wind, water, or ropes- such as various sounds, or songs of angels, and even the expression of words as by human voices; and those who made clocks which were run by wheels or by water or which measured time by means of the sun and distinguished the hours.Mechanics are those who make celestial spheres showing the various heavens and the movements of the planets and other heavenly bodies like a miniature universe, by the equal movement in rotation given by water power, as we are told was done by Archimedes of Syracuse, the first master.But the moving of very great weights with small force by means of diverse instruments and devices is the chief function of mechanics; thus do balances, steelyards levers, pulleys, wedges, mills, gears and smooth wheels, all sorts of screws, mangles, windlasses, augers, and many other things involving these.According to Aristotle, all these things reduce to the lever, the circle, and the round framework, which moves the more rapidly the larger it is.

The art of fortification of palaces and places, and of defending them, [an art] which may be called military architecture, is a mechanical profession, for with bastions and barricades and other defenses a man with few soldiers essays to repel many by means of machines and instruments and to maintain his advantage.Furthermore, the fabrication and operation of warlike instruments is the special province of this science, as, for example, catapults and slings and the like, which hurl fire and stones and masses of iron weighing 250 pounds to a great distance (as much as 300 paces, according to Silius and Vitruvius) with ruinous force; arrows and bolts as large as beams things that strike with damage nearby, such as rams, crowbars and sledges; maritime devices such as grappling hooks and rope ladders, bridges, floating towers, and similar ancient devices.These in turn have been replaced by artillery, which, causing frightful damage by means of a small amount of incendiary material, is itself governed by mechanical considerations.

This science, which embraces innumerable other uses, both pleasing and necessary to men, has in other times existed in various states of development in the hands of its practitioners.To begin with, in distant ages past, before the Trojan War, Dedalus of Athens was a great master mechanic; he was first to invent the saw and axe, the plumb line, the auger, the mast, sailyard, and sail, and other things; he designed the intricate Cretan labyrinth, and ultimately he tried to make himself and his son two pairs of wings, by which they might traverse the air like angels, as the poets sing.

One may believe that, in erecting the temple of Solomon, the greatest as to size, mastery of architecture, and ornamentation of any that has ever been made, and in constructing the pyramids and many other edifices of ancient times that have filled the world with amazement, excellent mechanics took part, raising on high immense stones and [doing] other things that such men attempt.Later came Eudoxus and Archytas, both worthy engineers.Of Archytas, we read that he made a wooden dove so masterfully adjusted and inflated that it flew through the air like a real dove.These men in turn were followed by Aristotle the philosopher, who propounded in writing a few very elegant questions of mechanics.Then came Demetrius the king, called the taker or destroyer of cities, because he made machines and devices by which he could descend on them from above and capture them.Perhaps these were similar to the machine called the Trojan Horse, which enabled the Greeks to take Troy, for Pausanias in his Attica says that he considers it madness to believe that this was really a horse, and not a machine designed to assault city walls and capture the city.It was this king who first raised mechanics to a degree of honor.

Archimedes, who was the best of all craftsmen up to his time in this profession and who is like a light that has since illuminated the whole world, brought the reputation of mechanics to a peak from the poor and vile art it had been (as Plutarch tells us in his life of Marcellus) and caused it to be numbered with the most noble and prized military arts.For when Marcellus attacked Syracuse by sea and by land with great Roman armies, Archimedes, by various machines and ingenuities, always repelled the forces, to their great shame and damage, as Livy, Plutarch, and others have narrated at length, naming his devices.And when Marcellus sailed near the walls to attack them with rams, good Archimedes with his derricks and iron claws lifted the ships into the air, releasing them to drop into the sea and sink, treating many ships thus, until the navies held back from approaching the walls.Nor did he cease to plague the enemy with this, but, as Galen notes in the third book of his Temperaments, and Giovanni Zonara and Tzetzes confirm, citing Diodorus and Dionysus, he built certain large concave mirrors in the right proportion for the distances of those ships from the walls, and, exposing these to the direct rays of the sun, he set the [ships] afire, as if miraculously.And when armies attacked by land, he struck them with other devices, so that neither at sea nor on land could the foe shield himself from the cleverness of that excellent mechanic, new defenses and terrible offenses forever appearing.Pappus of Alexandria cites the fortieth device of Archimedes, to show that his inventions numbered at least forty.Marcellus, seeing that nothing was to be gained by his attempted assaults and that his men were being exposed to danger simply by the existence of that valorous old man, came to share the opinion of his whole army that the defense of Syracuse was governed by divine power.Hence he changed the course of his warfare turning it into a siege and strictly preventing any foodstuffs from entering the city.

These, then, were the reasons for which mechanics rose to such glory that the Romans later honored it in their armies.Caesar took prisoner the chief of the smiths of Pompeii, called Magio Cremona; Vitruvius was made captain of catapults by Augustus Caesar, which would be equivalent to captain of artillery in our armies.This glory was afterward maintained by many eminent writers and masters of mechanics, such as Ctesibius of Alexandria, Hero of Alexandria, another Hero Athenaeus, Bion, and Pappus of Alexandria (who cites Carpus of Antioch), and by Heliodorus, Oribasius, and other Greeks who flourished at various periods.These men taught the theory, construction, and use not only of warlike machines but of all the others that belong to mechanics.Among the ancient Latins, Varro wrote of architecture and thus had to mention mechanics; Vitruvius, Vegetius, and some others spoke of the manufacture of military machines and machines to move weights, helping to sustain among men the reputation of mechanics.

But with the fall of the Roman Empire and the appearance of the barbarians in Italy, Greece, Egypt, and [places] where arts and letters had prevailed, nearly all the sciences declined miserably and were lost.Mechanics in particular was for a long time neglected.In war only slings, ballistae, crossbows, cranes, and a few such instruments were used until artillery, which had little by little fallen into disuse, came back.And as to the part of mechanics which deals with the moving of weights, very little understanding remained.Indeed, it appears that for a certain time the noblest arts and teachings, such as literature, philosophy, medicine, astrology, arithmetic, music, geometry, architecture, sculpture, painting, and above all mechanics, fell into dark shadow and lay entombed, and were later restored to light.As to mechanics, Jordanus Nemorarius, who wrote of the science of weights, began to resuscitate it somewhat, and then Leon Battista Alberti in his architecture; Tartaglia opened the road to many mechanical theorems; Victor Fausto in the Venetian arsenal showed himself a fine mechanic; the Reverend Monsignor Barbaro, Elector of Aquileia, in his commentary on the tenth book of Vitruvius, named the instruments used to move weights; Georgius Agricola in the sixth book of his Metals collected many machines for the raising of weights; and some others.Finally the author of the present work, proceeding in a very different manner from the others named, has taught in an admirable order and with true and certain reasoning, best among Latin writers, the whole science of moving weights.

Now just as the modern writers I have mentioned, and above all the author of the present book, have elaborated and elevated mechanics by their words and their books, so your Excellency has celebrated it and praised it in discourses and by means of actual operations, making familiar and domestic various machines, constructed on the most profound theory, and performing experiments in the moving of the greatest weights that can possibly be of use to man.So it may be truly affirmed that you, on the one hand, and the authors of these treatises, on the other, have restored to mechanics its pristine honor, which had been lost to it from ancient times to our own.

It is now about forty years since your Excellency proposed for diversion some forty problems in mechanics to Niccolo Tartaglia, a man much admired at his time in that profession; he delighted to solve subtle questions of mechanics and mathematics.In his dialogues he introduced many great persons as speakers, though sometimes he made them say things of which they were ashamed.Your problems were mostly difficult ones; some he attempted to solve, but from others he excused himself, saying that each of them would require a volume by itself, as one may read in his books on the new science.

Now it is no wonder that you penetrated so deeply into this matter, could work in mechanics so well, and were a master of the whole art of fortifying places and every other military matter, for you were raised by your distinguished father in the company of men learned in science and other affairs.Among these was Constantine Lascari, a noble and learned Greek, by whom you were instructed successfully in letters, arithmetic, geometry, astrology, and geography and were taught to design and work with your hands in various ways-to ride, to handle arms, to fire the arquebus and artillery pieces, to make gunpowder, and to practice the excellent art of the bombardier-to live soberly, and in work to tolerate heat, cold, and every discomfort.All of these are things that guide the spirit and harden the body to military enterprises.

At the age of sixteen, you were sent with twelve horsemen, mostly Turkish, and with sufficient funds to serve in the entire war in Italy from the capture of Francis I of France to the general peace which ensued in the year 1529.This war involved practically every known military movement, through the large armies which confronted one another, the quality and number of undertakings, and a thousand other important events and stratagems that took place; and above all because in one field or another and at all seasons the foremost soldiers of the world were fighting in great numbers with prudence, astuteness, and bravery, for the honor of conquering and being the victors ---

The Christian princes having returned in peace, you dedicated yourself to the service of your Serene Lords, where in the most important and greatest charges and in two wars you have added fifty years of splendid service to the two hundred of your Savorgnan predecessors; and during this time you have made some fifty great catapults in different provinces of their states, well thought out and masterfully made, with great economy of public funds.

Returning now to mechanics, I may mention that some years ago I visited your fortress at Osopo.There I was delighted to see your warehouse of arms neatly arranged, a magazine of warlike machines and machines to move weights, of which you have through your industry fabricated perhaps a dozen different sorts, some to drag weights, some to raise great weights with little force.One has but a single toothed wheel, yet it draws up steeply five of your cannons by the strength of Gradasso, your dwarf.Another, with but an ounce of force placed on the handle, sets in motion 114,000 pounds of weight; and since a man usually has 50 pounds of force in his arm, if this were used on the same handle, it is evident that the said machine would have the incredible power of moving more than eight million pounds.These machines can be carried by a mule, and some even by a man; they are necessary for various affairs, especially the handling and transporting of great artillery pieces---

Now in peace it has pleased your Excellency to investigate for your amusement many and various sorts of arrangements for the moving of weights, to utilize these in the construction of stone dams to hold back the force of the Tagliamento, that it might not damage the fields of Osopo, and to make use of them in time of war.Thus did Archimedes, according to Plutarch; for in time of peace, at King Hieron's request, he invented many machines for sport and as a geometrical pastime, and when war came, he was able to use these against the Romans.Various authors attest that he, seated at a certain machine (called tripaston according to Oribasius) which was operated by three ropes, drew from the sea to the land a great ship of the king's.With the power of his left hand he moved by means of this instrument a load of 5000 bushels. Computed at 45 pounds weight per bushel, the total came to 225,000 pounds; and he boasted he could move the earth if he had a place to rest his lever.As to that machine described by Pappus in the eighth book of his Collections, which had five wheels on axles and a worm gear with its handle, I am sure that you could design instruments to do as mech.

Having seen and tested these various devices at Osopo and having been shown by you for the first time this book, which you highly commended, I formed the idea that it would be useful to translate it into our native language, so that those might understand it and profit from it who have no knowledge of Latin.The work finished and printed, I send it to your Excellency, for you are master of this subject and encourage literature, which comes to naught unless favored (after God) by great gentlemen.If to some degree I shall by my labors bring something useful to lovers of mechanics, let them know that it is to you they owe this work to a large degree.

From your affectionate servant Filippo Pigafetta Venice, 28th June 1581

Preface of Filippo Pigafetta

To the Reader:-The present book contains six treatises, the first on the balance and steelyard, the second on the lever, the third on pulleys, the fourth on the windlass, the fifth on the wedge, and the last on the screw, all which are mechanical instruments.It is entitled Mechanics.But this word "mechanics" is perhaps not understood by everyone in its true sense, and some are even found who consider it an insulting word, for in many parts of Italy a man is called a mechanic in scorn and degradation, and in some places people are offended to be called even "engineer".Hence it will perhaps not be out of place to mention that "mechanic" is a most honorable term according to Plutarch, meaning business pertinent to military affairs, and is appropriate to a man of high position who knows how with his hands and his heart to carry out marvelous works of singular utility and delight to mankind.

To name some among the many philosophers and princes of past centuries, Archytas of Tarentum and Eudoxus the companion of Plato, whom Plutarch mentions in his life of Marcellus, were excellent engineers and mechanics; King Demetrius was a clever inventor of war machines and worked with his hands also; and among the Sicilian Greeks the most famous mechanic and engineer was Archimedes of Syracuse, who was of noble lineage and a relative of King Hieron of Sicily.

Although in the same work Plutarch affirms that Archimedes disparaged mechanics as base and vile and material and did not deign to write of it, and that he employed himself on machines not as a principal work but merely for amusement and as a geometrical game, requested by the king, yet we read in other authors that he wrote a book on the measurement and proportions of every kind of vessel, devising the shape of the great ship of Hieron, in which nothing was lacking.Pappus of Alexandria quotes from Archimedes' book on the balance, which is entirely mechanical; also in the eighth book of his Mathematical Collections he shows an instrument for the moving of weights, the fortieth invention of Archimedes, of which he said: "Give me a place to stand, and I shall move the earth." The mechanic Carpus wrote that Archimedes composed a book on the making of spheres, which is a mechanical task.Moreover, this same Archimedes himself more than once cites [mechanics] in his book on the Quadrature of the Parabola, with these words: "Since it is demonstrated in the Mechanics" referring to some propositions of his book on equiponderance, which is entirely mechanical.Also a part of his book on the Quadrature of the Parabola and the second book of his work on Bodies in Water are mechanical.From this it is seen that Archimedes not only performed mechanical works, but also wrote many treatises of it.Plutarch admits that Archimedes rose in reputation more from his mechanical undertakings than from any other teaching and, indeed, by means of these gained the fame not of human science but of divine wisdom.Hence one may ask why Plutarch allowed himself to say that Archimedes disparaged mechanics?Surely he would have been wrong to show little esteem for that which gained him much greater fame than any other science he possessed.

Among the Romans, Vitruvius was a good mechanic and served as captain of catapults and other war machines under Octavius Caesar; he wrote a book on architecture and made a fortune from it.

Hence to be a mechanic and an engineer after the example of these great men is not unworthy of a gentleman.Mechanics is a Greek word, meaning a thing made artificially to move, as by a miracle and beyond human power, when great weights are moved with small force; and in general it includes every structure, machine, instrument, windlass, mangle, or masterfully discovered device constructed for such effects, and many others in any science, art, and practice.I mention these concretely to put the matter in a form suited to the taste of most men, leaving accurate definition to a more appropriate time.

It should be added that under this general title the author has contented himself at present to teach (and he is the first Latin writer to do so)' by means of easy and plain demonstrations merely the method of understanding and operating the six mechanical instruments, to which all others may be reduced.For these are basic and fundamental, and there may be compounded in various ways combinations of two, three, or more; thus the windlass may be combined with the pulley, the screw with the windlass or the lever, and so on.This may be done at will by anyone who can proceed with good judgment in various works, as the author notes at the end of this volume.

Now although the author has reasoned of these machines in good method and admirable order, and nothing inherently obscure must be mastered, yet it requires a man's whole mind, and the demonstrations should be read attentively more than once with concentration.

The Book of Mechanics of Guido Ubaldo
Composed in Latin in 1577
Translated into Italian in 1581
With Commentaries by Filippo Pigafetta
DEFINITIONS

The center of gravity of any body is a certain point within it, from which, if it is imagined to be suspended and carried, it remains stable and maintains the position which it had at the beginning, and is not set to rotating by that motion.This definition of the center of gravity is taught by Pappus of Alexandria in the eighth book of his Collections.But Federico Commandino in his book On Centers of Gravity of Solid Bodies explains this center as follows:

The center of gravity of any solid shape is that point within it around which are disposed on all sides parts of equal moments, so that if a plane be passed through this point cutting the said shape, it will always be divided into parts of equal weight.

AXIOMS [Communes Notiones]

1. If, from things of equal weight, other things of equal weight be taken, the remainders will be of equal weight.2. If, to things of equal weight, other things of equal weight be added, the wholes will be of equal weight.3. Things equal in weight to the same thing are equal in weight to each other.

POSTULATES [Suppositiones]

1. Every body has but a single center of gravity.2. The center of gravity of any body is always in the same place with respect to that body.3. A heavy body descends according to its center of gravity.

On the Balance [Page 11]

[Figure 1]

Before a discussion of the [actual] balance, to make matters clear , let the straight line AB be a balance, with its support [trutina] CD, which in accordance with common practice is kept always perpendicular to the horizon.The stationary point C, about which the balance turns, is (though improperly) called the center of the balance, even if it is above or below the balance, and CA and CB [in the first diagram, or their equivalents in the others] are called the arms or distances of the balance.And if the center of the balance is above or below A B, let there be drawn from it a line at right angles [to A B] which sustains the balance and will be called the perpendicular; and however the balance may move, this will always remain perpendicular to it.

LEMMA

[Figure 2]

Let the line AB be perpendicular to the horizon , and describe the circle AEBD having the diameter AB and the center C. I say that the point B is the lowest place on the circumference AEBD, and the point A is the highest; and that any other points such as D and E, which are equidistant from A, are situated equally below it; and that those points which are closer to A are higher than those which are more distant from A.[The proof is omitted. F represents the center of the world].

PROPOSITION I

[Figure 3]

If the weight is supported at its center of gravity by a straight line, it will remain stationary only if that straight line is perpendicular to the horizon.

[The proof, omitted here, rests on Postulate 3.]From this it may be deduced that a weight supported at any point in any manner will never remain at rest unless the line drawn from the center of gravity to the point of support is perpendicular to the horizon.

[Figure 4]

Thus, let the weight be supported by the lines CG and CH.I say that the line BC being perpendicular to the horizon, the weight will remain at rest; but the line CF being not perpendicular to the horizon, the point F will move downward to D, where it will rest, and the line CD will be perpendicular to the horizon.All which may be shown by the foregoing reasons.

PROPOSITION II

[Figure 5]

A balance parallel to the horizon, with its center above [and] having equal weights at its extremities which are equidistant from the perpendicular [CD], when moved from this position and released, will return and rest in it.

[The proof, omitted, is based on Archimedes, On Piane Equilibrium, I. 4.]

PROPOSITION III

[Figure 6]

A balance parallel to the horizon, with its center below [and] having equal weights at its extremities which are equidistant from the perpendicular [CD], will be at rest- but if moved and left tilted, it will move toward the lower side.

[The proof, omitted, is based on Archimedes].

PROPOSITION IV

A balance parallel to the horizon, having its center within the balance and with equal weights at its extremities, equally distant from the center of the balance, will remain stable in any position to which it is moved.

[Figure 7]

Let the balance be the straight line AB, parallel to the horizon, with its center C in the line AB and the distance CA equal to the distance CB; and let the weights A and B be equal, and have their centers of gravity in the points A and B. Let the balance be moved to DE and left there.I say, first, that the balance DE will not move and will remain in that position.Now since the weights A and B are equal, the center of gravity of the combination of the two weights A and B will be at C.Hence the same point C will be the center of gravity of the balance and of the whole weight.And since the center of gravity of the balance, C, remains motionless while the balance AB together with the weights moves to DE, the center of gravity is not moved.Therefore the balance DE, being hung on this, will not move, by the definition of the center of gravity.The same likewise happens with the balance AB parallel to the horizon or in any other position.Hence the balance will remain where it is left; which was to be demonstrated.

Although we have considered in the foregoing only the weights of the bodies which are at the ends of the balance, without that of the balance itself, yet, since the arms of the balance are equal, the balance will behave the same whether we consider its weight together with those of the bodies or without them, for the same center of gravity without weights will be that of the balance alone.Likewise if the weights are attached to the ends of the balance, in the usual manner, it will be the same, provided that the lines drawn from where the weights are attached toward the center of heavy things (the balance being moved in any manner) go to meet in the center of the world, since, when the weights are attached in this manner, they bear down as if they had their centers of gravity in those same points.Whence we may consider the results in just the same way.

But with regard to this last conclusion, many things are said by men who believe otherwise.Hence it will be well to dwell further on this; and according to my ability I shall endeavor to defend not only my own opinion but Archimedes too, who seems to have been of the same opinion.

[Figure 8]

Things being as before, let there be drawn the line FG plumb to AB and to the horizon; and with the center C at the distance CA describe the circle ADFBEC.The points ABDE will be on the circumference, because the arms of the balance are equal.Now these authors are of the opinion that the balance DE does not move to FG, nor remain at DE, but returns to the line AB parallel to the horizon; I shall show that their opinion cannot stand.For if what they say is true, this result will occur because either the weight D is heavier than the weight E or the weights are equal but the distances at which they are placed are not equal; that is, CD does not equal CE, but is greater.But it is clear from our assumptions that the weights at D and E are equal, and that the distance CD is equal to CE.Now since they say that the weight placed at D is heavier in that position than is the weight placed at E in its lower position, then, when the weights are at D and E, the point C will no longer be their center of gravity, inasmuch as they would not be stable if suspended from C.But that center will be on the line CD, by Archimedes, On Plane Equilibrium, 1. 3.It will not be on CE, the weight D being heavier than the weight E; let it therefore be at H, from which, if they were suspended, the weights would remain stationary.And since the center of gravity of the weights joined by AB is at the point C, but that of those placed at D and E is the point H, when the weights A and B are moved to DE, the center of gravity C would be moved toward D and would approach closer to D, which is impossible.For the weights remain the same distance apart, and the center of gravity of any body stays always in the same place with respect to that body.And though the point C is the center of gravity of two bodies A and B, yet these being joined together by the balance, so that they are always- the same, the point C will be their center of gravity as if they were a single body, since the balance together with the weights makes a single continuous body whose center of gravity remains always at the center.Therefore the weight placed at D is not heavier than the weight at E.If they should say the center of gravity was not in the line CD, but must be in CE, the same error would follow.

Moreover if the weight D will move down, the weight E will move up.Therefore a weight heavier than E, put in the same place, will weigh the same as the weight D, and it will happen that unequal heavy bodies, placed at equal distances, will weigh equally.Add, then, to the weight E some heavy thing so made as to counterpoise D if suspended from C.But it having been shown above that point C is the center of gravity of equal weights placed at D and E, then if the weight E shall be heavier than D, the center of gravity will still be in the line CE; let this center be K.But by the definition of the center of gravity, if the weights were suspended from K, they would be stable.Hence if they were hung from C, they would not be stable, which goes against the assumption, for the weight E will move upward.For if they weighed equally when hung from C, it would come about that there would be two centers of gravity for one body, which is impossible [by the first axiom].Therefore the weight placed at E, heavier than that at D, would not weigh the same as D being hung from C.The equal weights placed at D and E, therefore, hung from their center of gravity, will weigh equally and will stay motionless; which was to be proved.

To this last contradiction these authors reply that it is impossible to add to E so small a weight that, if indeed they [E and D] were suspended from C, the weight E would not continue downward to G.But we have assumed this to be possible, and believe it can be done.For, the excess of weight D over weight E having some ratio and quantitative part, we imagined it to be not only minimal but also capable of infinite division.They seek in the following manner to prove that no such weight can be found, since it is not just minimal, but still less.

[Figure 9]

Things being taken as before, and from the points D and E the lines DH and EK being drawn perpendicular to the horizon, let there be taken another equal circle LDM, with center N, which is tangent to the circle FDG at the point D. NC will be a straight line, and, since the angle KEC is equal to the angle HDN, and the angle CEG is likewise equal to the angle NDM, being contained within equal radii and circumferences , the remaining mixed angle KEG equals that of HDM.Accordingly, they assume that the smaller the angle contained between the vertical line and the circumference, the heavier the weight will be in that position.So that, as the angle HDG contained between HD and the circumference DG is less than the angle KEG (that is, than the angle HDM) in this same proportion will the weight at D be heavier than it would be at E.But the ratio of angle MDH to HDC is smaller than any other ratio that exists between greater and smaller quantities; therefore the proportion of the weights at D and E will be the smallest of all possible ratios, or, rather, will not be a ratio at all.That the ratio of MDH to HDG is the least of all, they demonstrate by this necessary reason: that MDH exceeds HDG by a curvilinear angle MDG, which angle is less than any angle made by straight lines; and since no smaller angle can be found than MDG, the ratio of MDH to HDG will be the least of all ratios.

This reasoning seems very frivolous; for, though the angle MDG is less than any angle made by straight lines, it does not follow that it is the least of all possible angles; inasmuch as, if we draw the line DO from D perpendicular to NC, it will touch both the circumferences LDM and FDG at the point D.But since the circumferences are equal, the mixed angle MDO is equal to the mixed angle ODG.Hence one of the angles, that is, ODG, will be less than MDG; that is, less than the minimum.Therefore the angle ODH will be less than MDH; whence the ratio of ODH to HDG will be less than that of MDH to HDG.So there will be a ratio still less than the minimum, which we shall show further to be infinitely less in this manner.Describe the circle DR with its center at E and with radius ED; the circumference DR will touch the circumference DG at the point D and the line DO at the point D.Hence the angle RDG will be less than the angle ODG, and likewise the angle RDH less than the angle ODH --- And thus, if infinitely many circumferences are drawn between DO and DG, we shall find the ratio diminishing ad infinitum and it follows thus that the ratio of the weight placed at D to that at E is not so small that one infinitely less cannot be found.And since the angle MDG can be divided in infinitum so also one may divide in infinitum the excess of weight which D has over E.

Nor should it be omitted that they have assumed in their proof as a thing known that the angle KEG is greater than the angle HDG, which indeed is true if DH and EK are parallel.But since, as they likewise assume, the lines DH and EK meet at the center of the world, they are not ever parallel, and not only will the angle KEG not be greater than the angle HDG, but it will be smaller.

[Figure 10]

For example, draw the line FG to the center of the world S, and join DS and ES.The angle SEG is to be demonstrated less than the angle SDG.From the point E draw the line ET tangent to the circle DGEF, and from the same point draw EV parallel to DS.Then since EV and DS are parallel, ET and DO are parallel; the angle VET will be equal to the angle SDO, and the angle TEG to ODM, being contained between tangents and equal circumferences.Therefore the whole angle VEG will be equal to SDM.Take away from the angle SDM the curvilinear angle MDG, and from the angle VEG the angle VES; and the angle VES formed by straight lines is greater than the angle MDG formed by curved lines, so the remaining angle SEG is less than SDG.Hence by their own suppositions not only will the weight placed at D fail to be heavier than that at E, but on the contrary the weight at E will be heavier than that at D.

[Figure 11]

Nevertheless, they adduce reasons by which they attempt to show that the balance DE necessarily returns to AB, parallel to the horizon.First they show that a given weight is heavier at A than at any other place, and this position they call the " level position," the line AB being parallel to the horizon.Then the closer the weight is to A, the heavier it will be in comparison with any other position; that is, it will be heavier at A than at D, and at D than at L^.and similarly heavier at A than at N, and at N than at M, only one weight on one of the arms, moved up or down, being considered.For they say that, if the support of the balance is on CF, the weight placed at A is farther from the support than at D, and at D it is farther than at L; for when the lines DO and LP are drawn perpendicular to CF, the line AC is longer than DO, and DO than LP, and the same for the points N and M.They then say that the weight is heavier where it will move more swiftly, and it moves most swiftly of all from A; whence it is heaviest at A.Likewise, the closer it is to A, the more swiftly it will move; therefore it will be heavier at D than at L.

Next, they deduce another cause from the quality of straighter or more bent motion; that is, when the descent of a weight is more nearly straight along equal arcs, the weight appears to be heavier, because when it is free it moves naturally in a straight line.But at A it descends most straightly; therefore at A it will be heaviest, and they show this by taking the arc AN equal to the arc LD, drawing from the points N and L the lines NR and LQ parallel to the line FG, which they call the "line of direction"; and these cut the lines AB and DO at R and Q.From the point N is drawn the line NT perpendicular to FG; and they truly show LQ to be equal to PO, and NR to CT, and the line NR to be longer than LQ. Now since the descent of the weight from A to N along the circumference AN runs over a greater part of the line FG (which they call "partaking more of the straight") than [for example] the descent from L to D along the circumference LD , inasmuch as the descent AN runs over the line CT, while the descent LD covers PO s and CT is greater than PO, the descent AN will be "straighter" than the descent LD.Therefore the weight placed at A will be heavier than at any other place; and in the same way they show that the closer the weight is to A, the heavier it is; that is, the arcs LD and DA being equal, and the line DR being drawn from D perpendicular to AB, DR will be equal to CO; and thence they show that the line DR is greater than LQ, and they say that the descent DA partakes more of the straight than does LD, because the line CO is longer than OP.Hence the weight will be heavier at D than at L, and likewise at N than at M. Hence the assumption by which they show the balance DE to return to AB they declare as a thing known and manifest; that is, that the weight is positionally heavier in proportion as its descent from a given place is less bent, and they say the reason for the return [of the balance to the level position] is this: the descent of the weight placed at D is straighter than its descent at E, since the weight at E partakes less of the line of direction in descending than does the weight D.For if the arc EV is equal to DA, and the lines VH and ET are drawn perpendicular to FG, DR will be greater than TH. Consequently, by their assumption, the weight at D is positionally heavier than at E.Hence the weight at D, being heavier , will move downward, and the weight at E upward, until the balance DE returns to AB.

Still another reason for this return is that, when the support of the balance DE is from above, the line CG is the meta, and, since the angle GCD is greater than the angle GCE, and the angle greater than the meta renders the weight heavier, then when the support of the balance is from above, the weight at D will be heavier than at E, and thus D will return to A, and E to B.

And by the above arguments they attempt to show that the balance DE returns to AB, which arguments seem to me to be easily answered as follows.

First, when they say that the weight placed at A is heavier than in any other position, they deduce this from its varying distances from the line FG, the swiftest and straightest movement being from point A.To begin with, they do not truly demonstrate that the weight moves more swiftly from A than from any other place, nor does it follow that, since CA is greater than DO, and DO than LP, the weight placed at A is heavier than that at D, and at D than at L.Now the intellect is not satisfied unless this can be demonstrated from some other cause, for this appears to be merely a sign rather than a cause.The same is true of their other argument, adduced from movements being straighter or more bent.Besides, all the things adduced from swifter and slower movement to persuade us that the body at A is heavier than that at D do not show that the weight at A, by its being at A, is heavier than the weight at D, by its being at D, but only by their departing from the points D and A.So, before going further, I shall first show that the closer a weight gets to the line FG, the less it weighs, both as to its position and as to its departure therefrom; and at the same time I shall show it to be false that the weight is heaviest at A of all places.

[Figure 12]

Draw FG to the center of the world, S, and from S draw also a line tangent to the circle AFBG.This line cannot be drawn from S to touch the circle at A, inasmuch as, if the line AS were drawn, the triangle ACS would have two right angles, that is, SAC and ACS, which is impossible.Still less can it touch the circle at A in the quadrant AF, for it would cut the circle.Therefore it will touch below, and let this line be SO; then add the lines SD and SL, which cut the circumference AOG at points K and H; and join also CK and CH.And thus the closer the weight is to F, the higher it stands above the center, as the weight at D presses more on and stands higher above the turning point C as center; that is, the weight at D weighs down more on the line CD than it would at A on the line CA, and still more at L on the line CL.For, the three angles of each triangle being equal to two right angles, and the angle DCK of the isosceles triangle DCK being less than the angle LCH of the isosceles triangle LCH, the angles CDK and CKD taken together are greater than CLH plus CHL; and the half of this, that is, CDS, will be greater than CLS.And CLS being lesser, the line CL approaches more closely to the natural movement of the weight placed at L when entirely free, that is to say, to the line LS, than CD to the movement DS.For the weight placed at L would move freely toward the center of the world along LS, and the weight at D along DS.But since the weight at L weighs wholly on LS, and that at D on DS, the weight at L will weigh more on the line CL than that at D on DC.Therefore the line CL will more sustain the weight than the line CD; and in the same way, the closer the weight is to F, it will be shown for this reason to be more sustained by the line CL, since the angle CLS is always less, which is obvious.For if the lines CL and LS should come together, which would happen at FCS, then the line CF would sustain the whole weight that is at F and would render it motionless, nor would it have any tendency to descend [gravezza] along the whole circumference of the circle.Therefore the same weight, by diversity of position, will be heavier or lighter, and this not because by reason of its place it sometimes truly acquires greater heaviness and sometimes loses it, being always of the same heaviness wherever it is, but because it presses [grava] more or less On the circumference, as at D it presses more on the circumference DA than at L on the circumference LD.That is, if the weight shall be sustained [jointly] by the circumferences and the straight lines, the circumference AD will more sustain the weight placed at D than the circumference DL sustains the weight placed at L, for CD helps less than CL.Besides this, if the weight at L were completely free, it would move down along LS were it not prevented by the line CL, which forces the weight at L to move beyond the line LS along the circumference LD and in a certain sense pushes it, and, in pushing it, comes partly to sustain it; for, if it did not sustain it and give it resistance, the weight would move down along the line LS, rather than along the circumference LD.Similarly CD offers resistance to the weight placed at D, forcing it to move along the circumference DA.In the same way, the weight being at A, the line CA will constrain it to move outside the line AS along the circumference AO, for the angle CAS is acute, ACS being a right angle.Therefore the lines CA and CD to some degree, though not equally, offer resistance to the weight, and whenever the angle at the circumference of the circle made by the line coming from the center of the world S and that from the center C shall be acute, we shall prove the same thing to occur.Now since the mixed angle CLD is equal to the angle CDA, being contained by radii and the same circumference, and the angle CLS is less than the angle CDS, the-remainder SLD will be greater than the remainder SDA.Hence the circumference DA, which is the path of descent of the weight at D, is closer to the natural movement of the free weight at D (that is, the line DS) than the circumference LD is to the line LS.Therefore the line CD will offer less resistance to the weight placed at D than the line CL to the weight placed at L.So the line CD will sustain less than CL, and the weight will be more free at D than at L, being moved more naturally along DA than along LD.Whence it will be heavier at D than at L.Similarly we shall demonstrate that CA sustains less than CD, and that the weight at A is more free and heavier than at D.Next, in the lower part, for the same reasons, the closer the weight is to G, the more it will be retained, as at H by the line CH than at K by the line CK; for, the angle CHS being greater than the angle CKS, the lines CH and HS approach closer to the [line of] direction than CK and KS, and hence the weight will be more retained by CH than by CK; for, if CH and HS meet in a line, as happens when the weight is at G, then the line CG would sustain the whole weight at G, so that it would remain motionless.Therefore the smaller the angle contained between the line CH and the line of the weight in free fall (that is, between CH and HS), the less the line CH will retain the weight; and where it is less retained, it will be freer and heavier.Besides which, if the weight were free at K, it would move along the line KS; but it is impeded by the line CK, which forces the weight to move from the line KS along the circumference KH.This restricts it in a certain way and thus comes to sustain it; for, if it were not sustained, it would move along the straight line KS and not along the circumference KH.Similarly CH retains the weight, constraining it to move along the circumference HG.And since the angle CHS is greater than the angle CKS, if we take away the equal angles CHG and CKH, the remainder SHG will be greater than the remainder SKH.Hence the circumference KH (that is, the descent of the weight placed at K) will be closer to the natural movement of the free weight placed at K (that is, to the line KS) than the circumference HG is to the line HS.Hence the line CK retains less than CH, the weight moving more naturally by KH than by HG.With similar reasons it will also be shown that the smaller the angle SKH, the less the line CK will sustain.The weight therefore being at O, since the angle SOC not only is less than the angle CKS but is the least of all angles that come from the points C and S and have their apex on the circumference OKG, the angle SOK will be less than the angle SKH and less than the others so formed.Hence the descent of the weight placed at O will be closer to the natural movement of a free weight at O, than if placed at any other position on the circumference OKG, and the line CO will sustain the weight less than if it were at any other place on the circumference OG.Likewise, since the angle of contact SOK is less than the [mixed] angle SDA, or SAO, or any other such angle, the descent of the weight placed at O will be closer to the natural movement of this weight than if placed at any other site along the circumference ODF. Besides, the line CO cannot push the motion of the weight placed at O when it moves down so that it will move outside the line OS, as the line OS does not cut the circle, but touches it, and the angle SOC is a right angle and not acute; hence the weight placed at O will never weigh against the line CO, nor will it bear upon the center, as would happen at any other point above O.Hence the weight placed at O will for this reason be free, and more completely so at this point than at any other in the circumference FOG; and thus it will be heavier and will bear down more here than elsewhere.And the closer it is to O, the heavier it will be than anywhere farther away.And the line CO will be parallel to the horizon, though not to the horizon of the point C (as they believe), but rather to that of the weight placed at O; for the horizontal must be taken from the center of gravity of the body.All of which was to be shown.

[Figure 13]

But if the balance arm were greater than CO, say, by the amount CD, the weight placed at O would likewise be heavier. Describe the circle OH, with center D and radius DO.The circle OH will touch the circle FOG at the point O and will also touch the line OS at that same point, this being the straight and natural descent of the weight placed at O.And since the angle SOH is less than the angle SOG, the descent of the weight placed at O along the circumference OH will be closer to its natural movement OS than would that along the circumference OG.Hence the weight at O will be freer, and consequently heavier, than at C (the center of the balance being at D).Similarly it will be shown that the longer the arm DO, the heavier will be the weight placed at O.

[Figure 14]

But if the same circle AFBG with its center R shall be closer to the center of the world S, and if a line ST is drawn from the point S tangent to the circle, the point T (where the weight is heaviest) will be farther from the point A than is the point O.Draw the lines OM and TN from the points O and T, plumb to CS, and add RT, the center R being in the line CS, and the line ARB being parallel to ACB.Then, the triangles COS and RTS being right triangles, SC will be to CO as CO is to CM.Similarly, SR is to RT as RT is to RN.Now, RT being equal to CO, and SC greater than RS, the ratio of SC to CO will be greater than that of SR to RT; whence, likewise, CO has a greater ratio to CM than RT to RN.Thus CM will be less than RN.Then cut RN at P so that RP shall equal CM, and from the point P draw the line PQ parallel to the lines MO and NT, [such that] it will cut the circumference A T at Q; and finally join R and O.Now since CO and CM are equal respectively to RQ and RP, and the angle CMO is equal to the angle RPQ, the angle MCO is equal to the angle PRQ.But the angle MCA is equal to the angle PRA, both being right angles; hence the remainder OCA is equal to the remainder QRA, and the circumference OA is likewise equal to the circumference QA.Thus the point T, being farther from the point A than is Q, will also be farther from the point A than is the point O.Likewise it may be shown that, the closer the circle is to the center of the world, the farther T will be from A.Hence, as before, it may be shown that the weight on the circumference TAF will stand upon the center R, while on the circumference TG it will be held by the line, and it will be found heaviest at the point T.

[Figure 15]

And if the point G were the center of the world, then the closer the weight was to G, the heavier it would be; and hence wherever else the weight is placed than at G, it will always get support from the center C; for example , at K.Draw the line GK, along which the natural motion of the weight would be made; this will make an acute angle with the arm of the balance KC, because the base angles (at K and G) of the isosceles triangle CKG are always acute.

Now if the weight at K is compared with that at D, the weight at K will be heavier than that at D; for the line DG being drawn, and the three angles of any triangle being equal to two right angles, and the angle DCG of the triangle CDG being greater than the angle KCG of the isosceles triangle CKG, the base angles DGC and GDC taken together will be less than the angles KGC and GKC taken together; and half the sum, that is, the angle CDG, will be less than the angle CKG.Now since the weight at K would move in natural freedom along KG, and the weight at D along DG-these being the lines by which they are brought to the center of the world-the line CD, that is, the balance arm, will approach more nearly to the natural movement of a free weight at D, that is, to the line GD, than CK to the movement made along KG.Therefore the line CD will offer more support than CK, and therefore the weight at K will be heavier, by what has been said, than at D. Besides which, if the weight placed at K were entirely free, it would move down along KG if it were not impeded by the line CK which forces the weight to move beyond the line KG along the circumference KH; the line KG will sustain the weight in part, and will make resistance to it, forcing it to move along the circumference KH.And since the angle CDG is less than the angle CKG, and the angle CDK is equal to the angle CKH, the remaining angle GDK will be greater than the remaining angle GKH.Therefore the circumference KH will be closer to the natural free movement of the weight placed at K, that is, to the line KC, than the circumference DK to the line DG.Whereby the line CD makes more resistance to the weight placed at D than the line CK to the weight placed at K.Therefore the weight placed at K will be heavier than at D.Similarly it would be shown that the closer the weight was to F (as at L) the less it would weigh, but the closer it is to G (as at H) the heavier it is.

[Figure 16]

And if the center of the world were at S, between the points C and G, first it will be shown in the same way that the weight, wherever it is (as at H), gets support from the center C.For the lines HG and HS being drawn, the angle at the base GHC of the isosceles triangle CHG is always acute; whereby also SHC, being less than this, is also acute.But drawing from the point S the line SK plumb to CS, I say that the weight is heavier at K than at any other place in the circumference FKG, and the closer it shall be to F, or to G, the less it will weigh.Take the points D and L toward F, and join LC, LS, DC, and DS, and extend the lines LS, DS, KS, and HS to the circumference of the circle at E, M, N, and O, and join CE, CM, CN, and CO.Now since LE and DM come together at S, the straight angle LSE will be equal to the straight angle DSM.And as is LS to DS, so will SM be to SE; but LS is greater than DS, and SM than SE.Therefore LS and SE taken together will be greater than DS and SM, and for the same reason KN will be shown to be less than DM. Moreover, since the straight angle OSH is equal to KSN, by the same reason HO will be greater than KN.And in the same way KN will be shown to be less than any other line passing through S.And since of the two isosceles triangles CLE and DCM the sides LC and CE are equal to the sides DC and CM, and the base LE is greater than DM, the angle LCE will be greater than the angle DCM.Whence the base angles CLE and CEL taken together will be less than the angles CDM and CMD half the sum, that is, the angle CLS, will be less than the angle CDS.Therefore the weight at L will weigh more on the line LC than that at D will on DC, and will be more supported by the center C at L than at D. Similarly it will be shown that the weight at D will be more supported by the center C than at K.Therefore the weight at K will be heavier than at D, and at D than at L; and for the same reason, since KN is less than HO, the angle CKS will be greater than the angle CHS.Whereby the weight at H will be more supported by the center C than at K, and in this manner it will be shown that, wherever the weight is along the circumference FDC, it will be less supported by the center at K than if placed at any other point, and the closer it is to F or to C, the more it will be supported.Then since the angle CKS is greater than CDS, and CDK is equal to CKH, the remainder SKH will be less than the remainder SDK; whereby the circumference KH will be closer to the straight natural movement of the free weight at K, that is, to the line KS, than the circumference DK to the movement DS.Hence the line CD offers more resistance to the weight at D than CK does to the weight at K, and for this reason the [mixed] angle SHC will be shown to be greater than SKH, and consequently the line CH offers more resistance to the weight at H than CK to the weight at K.Similarly it would be shown that the line CL sustains the weight more than CD, and for the same reasons it will be proved that the weight at K will weigh less on the line CK than at any other place along the circumference FDC; and the closer it is to F or to C, the less it will weigh. Therefore it will be heavier at K than at any other place, and it will be less heavy the closer it is to F or to G.

Finally if the center C is the center of the world, it is manifest that the weight placed anywhere [on the circumference] will remain fixed Thus [if the weight is] placed at D, the line CD will sustain the whole weight, being vertical to that weight at D.Therefore the weight will remain at rest.

Now in the things demonstrated thus far, we have made no mention of the weight of the arms of the balance.If we next consider the weight of such an arm, we can find the center of gravity of the magnitude made by the weight and the arm; and circumferences can be described according to the distance from the center of the balance to this center of gravity, as if this contained the weight (which indeed it does).And the things we have found without considering the weight of the arm of the balance can be found in just the same way by considering this weight also.

[Figure 17]

From the things said, if we consider the balance to be removed from the center of the world as these other men have done (and as it is in fact), then it is clearly false for them to say that the weight is heavier at A than at any other place.And it is also false that the farther the weight is from the line FG, the heavier it is; for the point O is closer to FG than the point A

, the line drawn plumb from O to FC being less than CA.It is likewise false that the weight moves more swiftly from the point A than from any other place, for it will move more swiftly from the point O than from A, since at O it is more free than at any other place, and its descent from O will be closer to its straight natural movement than any other descent.

[Figure 18]

Besides this, when they argue by means of the straighter or more curved descent that the weight is heavier at A than at D, and at D than at L, they are certainly wrong; for if any weight were placed at any point on the circumference, as at D, its true descent would be made along the straight line DR parallel to FC, according to its natural movement, as was first said. For if a weight is placed anywhere, and we regard its natural movement to that proper place to which it moves straight by nature, taking into account the shape of the whole universe, then the space through which it moves naturally will always be along the line drawn from the circumference to the center.Therefore the natural straight descents of any free weight cannot be made by parallel lines, since all the lines meet in the center of the world.These men assume next that the motion of a weight from D to A along a straight line toward the center of the world is the same quantity as that from O to C, as if the points A and C were equally distant from the center of the world, which is likewise false---.Thus the assumption from which they demonstrate that the balance DE returns to AB turns out to be false, and all their demonstrations fall.Of course they might say that, because of our great distance from the center of the world, these differences are imperceptible, and by reason of that fact may be assumed void, as all those who have treated these matters have assumed, especially since their being imperceptible does not alter the fact that the descent of the weight from L to D partakes more (to use their own phrase) of the straight than the descent DA.Likewise the arc DA will partake less of the straight than the circumference EV, whence [they may say] the supposition will be true, and the other demonstrations will retain their strength.We may even concede that the weight will be heavier at A than [it will] anywhere else, and that the straight descent of the weight must be along a straight line parallel to FG, and that any points taken in straight lines parallel to the horizon are equally distant from the center; but it will not follow from this that their demonstration is true when they say that the weight is heavier at A than elsewhere, say, at L.For if it were true that the straighter a weight descended in this sense, the heavier it would be, then it would also follow that where the same weight would descend along equal arcs partaking equally in the straight, it would have equal weights; but this may be shown to be false in the following manner.

[Figure 19]

Let there be the equal arcs AL and AM, and join L and M, cutting AB at X; let LM be parallel to FG and perpendicular to AB, and XM will be equal to XL.If therefore the weight shall move from L to A along the circumference LA, its straight movement will be measured by the line LX.But if it moves g from A to M along the circumference AM, its straight movement will be measured by the line XM.Hence the descent from L to A will be equal to that from A to M, by reason of the equality of arcs as well as the equal straight lines perpendicular to AB.Therefore the weight at L will weigh the same as at A, which is false; for it is far heavier at A than at L.

And although AM and LA partake equally of the straight according to these men, perhaps they would say that because the first part of the descent from L, say LD, partakes less of the straight than the first part of the descent from A, that is, AN, the weight will be heavier at A than at L.For the arc AN being equal to LD, as was assumed, it partakes (according to them) of the straight CT, but LD partakes of the straight PO; hence the weight will be heavier at A than at L.But if this were true, it would follow that the same weight at the same place, merely considered in a different way with respect to that place, would be heavier and lighter, which is impossible.That is, if we considered the descent of the weight at L with respect to its descent from L to A, it would be heavier than if we considered its descent only to D.Nor can they deny from their own statements that the descent of the weight from L to A partakes of the straight LX, or PC, and that similarly the descent AM partakes of the straight XM, taking these also in this sense, as indeed one must take them; for if they want to show, by comparing the descent of the weight at D with that at E, that the balance DE returns to AB, they must show that the straight descent OC corresponding to the arc DA is greater than the straight descent TH corresponding to the arc EV.For if they should take but a part of the whole descent from D to A, such as DK, and show that the descent DK partakes more of the straight than the equal portion of the descent from the point E, it would follow that the weight at D, according to them, would be heavier than the weight at E, and would move only down to K, so that the balance would move to KI.Likewise, if they wished to show, by taking a part of the descent from K to A, such as KS, that the balance KI would return to AB, and [they then] showed that KS partook more in the straight than the equal descent from the point I, it would follow in the same way that the weight would be heavier at K than at I, and would move only to S.And, once more, if they showed that a portion of the descent from S to A (and so on) were straighter than the equal descent of the opposed weight, it would always follow that the balance SI would get closer to AB, but they could never show that it arrived there.Hence if they want to show that the balance DE returns to AB, they must assume that the descent of the weight from D to A par takes of the straight by the quantity of the line drawn from D to AB at right angles; and thus, if we compare the equal descents DA and AN, which partake of the straight by OC and CT, it will turn out that the same weight weighs equally at D and at A, but if we take only the portion DA, it will be heavier at A than at D.Thus from a mere diversity in manner of consideration, and not from the nature of the thing, it would come about that the same weight was heavier or lighter.Moreover, their assumption does not affirm that the positional weight will be greater when at the same place the commencement of the descent is less oblique.Hence the postulate [they] adopted above, that is, that the weight is positionally heavier according as the descent from the same place is less oblique, is not to be conceded at all, for the reasons we have given; and not only that, but it is not difficult to show the exact opposite; that is, that the less oblique the descent of the same weight along equal arcs, the less it weighs.

[Figure 20]

Let there be as before the equal arcs AL and AM, and the point L close to F, and join L and M perpendicular to AB, and LX will also I be equal to XM.Then take the point P close to M, between M and G, and let the arc PO be equal to the arc AM; the point O will then be close to A.Now draw the lines CL, CO, CM, CP, and OP, and from the point P draw PN perpendicular to OC.Since the arc AM is equal to the arc OP the angle ACM will be equal to the angle OCP; but the right angle CXM is equal to the right angle CNP; therefore the remaining angle XMC of the triangle MXC will be equal to the remainder NPC of the triangle PCN.But the side CM is equal to the side CP; therefore the triangle MCX is equal to the triangle PCN and the side MX is equal to the side NP, so the line PN will be equal to LX.Draw from the point O the line OT parallel to A C, which shall cut NP at V, and also from the point P draw a line perpendicular to OT.This certainly cannot fall between O and V, the angle ONV being a right angle, making OVN an acute angle; wherefore OVP must be obtuse.Therefore a line drawn from the point P between O and V will not be perpendicular to OT; otherwise one of two angles of the triangle would be a right angle and the other obtuse, which is impossible.Hence the perpendicular will fall on the line OT in the segment VT; let it be PT, whence PT will be the direction of descent for the arc OP.Now since the angle ONV is a right angle, the line OV will be longer than ON, whence OT will likewise be greater than ON. Thus the line OP being drawn under the right angles ONP and OTP, the square of OP will be equal to the sum of the squares ON and NP, and likewise equal to the sum of the squares of OT and TP.Whence the sum of the squares of ON and NP will be equal to the sum of the squares of OT and TP.But the square of OT is greater than the square of ON, the line OT being longer than ON.Therefore the square of NP will be greater than the square of TP and thus the line TP will be less than the line PN and the line LX.Therefore the descent along the arc LA will be less oblique than along the arc OP.Hence according to what these men say, the weight at L will be heavier than at O, which is obviously false from what we have said above, inasmuch as the weight placed at O is heavier than at L.Therefore it is not possible to deduce from the degree of straightness or bending of the motion (taken in their sense) that the weight is positionally heavier according as, at a given place, the fall is less bent.And from this arises most of their error and delusion in this matter.And though at times the truth may accidentally follow from false assumptions, nevertheless it is the nature of things that from the false the false generally follows, just as from true things the truth always follows.So it is no wonder that, when they assume false things as true and use these as a basis, they deduce and conclude things that are quite false.These men are, moreover, deceived when they undertake to investigate the balance in a purely mathematical way, its theory being actually mechanical ; nor can they reason successfully without the true movement of the balance and with out its weights, these being completely physical things, neglecting which they simply cannot arrive at the true cause of events that take place with regard to the balance.

Besides this, even if we concede their assumption, they are far from the true theory of the balance when they argue from it that the balance DE must return to AB; for they always take one weight separately at D, or E, as if now one and now the other were placed in the balance, but never both of them together.

[Figure 21]

Indeed we must do quite the opposite; nor may we consider directly one weight without the other when we reason about them as placed in the balance.For when they see that the descent of the weight placed at D is less bent than that of the weight placed at E, the weight at D by their assumption must be heavier than the weight placed at E; and by being heavier (they believe) it necessarily moves downward and the balance DE returns to AB.This argument is of no use whatever.In the first place they always argue as if the weights at D and E must descend, considering the descent of one only without its being joined with the other.Ultimately, by a comparison of the descents of the weights [separately], they nevertheless conclude that the weight placed at D moves down, and the weight at E moves up, when the two weights are joined together in the balance.But from the same principles that they use, and from their demonstrations, one might equally well deduce the opposite of that which they have labored to defend.For if they compared the descent of the weight placed at D with the rise of the weight placed at E, along the lines EK and DH perpendicular to AB, the angle DCH being equal to the angle ECK and the right angle DHC equal to the right angle EKC and the side DC equal to the side CE, the triangle CDH will be equal to the triangle CEK and the side DH equal to the side EK; and, the angle DCA being equal to the angle ECB, the arc DA will also be equal to the arc BE.Therefore the weight placed at D descends along the arc DA, while the weight placed at E rises along the arc EB equal to DA, and the descent of the weight placed at D will (according to their practice) share in the straightness of DH, and the rise of the weight E will share the straightness of EK, equal to DH.Therefore the descent of the weight placed at D will be equal to the rise of the weight placed at E, and whatever the inclination of the one is to downward movement, such will also be the resistance of the other to upward movement.That is, the resistance to force of the weight placed at E in its ascent opposes itself to the natural power of the weight placed at D, because of their equality, so that by however much the weight placed at D goes with its natural power more swiftly downward, by so much the weight placed at E is more slowly forced upward.So that neither of the two will weigh more than the other; there being no action that proceeds from equality, the weight placed at D will not move the weight placed at E upward, because, if it did, it would be necessary that the weight placed at D should have stronger force in descending than should the weight placed at E in rising.But these things are equal; therefore the weights will remain at rest and the weighing down of the weight placed at D will be equal to the weighing down of the weight placed at E.

[Figure 22]

Moreover, they assume that the farther the weight is distant from the line of direction FC, the heavier it is.Now draw from the points D and E the lines DO and El perpendicular to FC; and, as before, the triangle CDO will be demonstrated to be equal to the triangle CEI, and the line DO to be equal to the line El.Therefore the weight placed at D is the same distance from the line FC as the weight placed at E; hence from their reasons and their assumptions the weights placed at D and E are equally heavy.And what is there to prevent our demonstrating that the balance DE will necessarily move to FC for a like reason?In the first place it may be deduced from their own demonstrations that the rise of the weight from E toward B is straighter than the rise of the weight D toward F; that is, the rise of the weight placed at D has less straightness than the rise of the weight placed at E, the arcs being equal.Thus it is assumed that the weight is positionally lighter to the extent that in a given place its rise is less straight.This assumption seems as evident as theirs, because the rise of the weight placed at E is straighter than the rise of the weight placed at D, so that by their assumption the weight placed at D will be lighter than the weight placed at E.Hence the weight placed at D will move up with respect to the weight placed at E in such a way that the balance will come to FC, and thus it would be demonstrated that the balance DE will move to FC, which demonstration is completely frivolous and suffers from the same fault.For although it may be conceded as true that the weight placed at E in rising will be heavier than the weight at D similarly rising, it does not follow from this that the weight placed at E in descending is heavier than the weight placed at D in rising.Therefore neither of these two demonstrations, which say that the balance DE returns to AB or that it moves to FC, is true.

In addition to this, if we shall examine their assumption and the force of their argument, we shall certainly see that these have a different meaning.For since the space through which the weight moves naturally must be from the center of gravity of this weight toward the center of the world, along a straight line drawn from the center of gravity to the center of the world, it will be said that a descent of the weight made in this way will be more or less oblique according to the space designated, and that it will move more or less along the said line, always going to seek its natural place by the closest route .Thus the descent is said to be more oblique, the more it departs from that space, and straighter the more it approaches it. Now in this sense the assumption need not give rise to difficulty on the part of anyone, because this is so clear in its truth and its agreement with reason that it does not appear to need to be made evident in any way.

Therefore if the free weight located at D must move to its proper place, and if S is taken as the center of the world, it will doubtless move along the line DS; similarly the free weight placed at E will move along the line ES.Now if (as is indeed the case) the descent of the weight is to be called more or less oblique according to its departure from or approach to the routes designated by the lines DS and ES, it is clear that, with regard to their natural movements toward their proper places, the descent of E along EC is less oblique than that of D along DA, it having been demonstrated above that the angle SEC is less than the angle SDA.Whence the weight at E will weigh more than at D, which is completely contrary to that which they have made such an effort to prove.

[Figure 23]

Now they may rise up against us, arguing as follows: If the weight placed at E is heavier than the weight placed at D, the balance DE will never remain in that position, as we have undertaken to maintain, but it will move to FC.To which we reply that it makes a great deal of difference whether we consider the weights separately, one at a time, or as joined together; for the theory of the weight placed at E when it is not connected with another weight placed at D is one thing, and it is quite another when the weights are joined in such a way that one cannot move without the other.For the straight and natural descent of the weight placed at E, when it is without connection to another weight, is made along the line ES ; but when it is joined with the weight D, its natural descent will no longer be along the line ES, but along a line parallel to CS.For the combined magnitude of the weights E and D and the balance DE has its center of gravity at C, and, if this were not supported at any place, it would move naturally downward along the straight line drawn from the center of gravity C to the center of the world S until C reached S.Therefore the balance DE together with its weights will move downward in such a way that the point C moves along the line CS until C arrives at S and the balance DE at HK, the balance HK having the same position that it had before; that is, HK is parallel to DE.Therefore join D to H and E to K.It is evident that when the balance DE moves to HK, the points D and E will move along the lines DH and EK, equal and parallel to each other and to CS.Hence if we regard the weights placed at D and E with respect to their conjunction, their natural movement is not along the lines DS and ES but along LDH and MEK, parallel to ES.But the natural inclination of a free weight at E will be along ES, and that of a similar free weight at D will be along DS.And therefore it is not contradictory that the same weight, now at E and now at D, is heavier at E than at D.But if the weights at E and D are joined together and we consider them with respect to their conjunction, the natural inclination of the weight placed at E will be along the line MEK, because the weighing down of the other weight at D has the effect that the weight placed at E must weigh down not along the line ES, but along EK.The same is true of the weight at E; that is, the weight at D does not weigh down along the straight line DS, but along DH, both of them being prevented from going to their proper places.Therefore, since the natural straight descent of weights placed at D and E is along LDH and MEK, their natural straight rise will likewise be along the same lines HDL and KEM, and the natural rise of the weight placed at E will be more or less bent, according as the route shall be more or less close to the line MK.And in exactly this way one must take both the rise and the descent of the weight at D according to the line LH.Therefore if the weight at E moves downward along the line EG, the weight at D would move upward along DF.And since the angle CEK is equal to the angle CDL, and the angle CEG is equal to the angle CDF, the remaining angle GEK will be equal to the remaining angle LDF.Now assuming that the weight is positionally heavier to the degree that its descent from a given place is less oblique, one will also admit without doubt that the weight will be positionally heavier according as its rise at a given point will be less oblique, since this is no less evident or agreeable to reason.Therefore the descent of the weight at E will be equal to the rise of the weight at D, because the descent of the weight at E has as much of the oblique as does the rise of the weight at D; and whatever may be the inclination of the one to downward movement, this likewise will be the resistance of the other to upward movement.Hence the weight at E will not move the weight at D upward, nor will the weight at D move downward in such a way as to raise the weight at E.For, the angle CEB being equal to CDA and the angle CEM equal to the angle CDH, the remainders MEB and HDA will be equal.Thus the descent of the weight at D will be equal to the rise of the weight at E, and the weight at D will not raise the weight at E.From which it follows that the weights at D and E, considered in conjunction, are equally heavy.

[Figure 24]

Now the second reason with which they attempt to show that the balance DE returns to AB is that, when the support of the balance is CF, its goal [meta] is CG, and, since the angle DCG is greater than the angle ECG, the weight placed at D will be heavier than that placed at E; therefore the balance DE will return to AB.In my opinion this does not follow, and this fiction about the support and the goal should just be left out and passed over in silence; for to say anything about it only confuses the issue, the whole thing being arbitrary, since no necessary reason why the weight placed at D at the larger angle will be heavier, or why the greater angle is the cause of greater weight, is given anywhere.The angle GCD being equal to the angle FCE, then if the angle GCD is the cause of heaviness, why is not the angle FCE similarly such a cause? For this effect they attempt to adduce the following reasoning: Since CG is the goal and CF the support, if (they say) CG were the support and CF the goal, then the angle FCE would be the cause of heaviness, and not DCG which is equal to it .This reasoning is sheer imagination and quite arbitrary.For what can it matter whether the support is CF or CG, when the balance DE is always sustained at the same point C?But let us make their delusion still more obvious.

[Figure 25]

Let there be the balance AB with center C and support FG which remains motionless and sustains the balance AB at the point C.Now let the balance move to DE; and since the support is both above and below the balance, what angle will be the cause of heaviness, the balance DE being sustained always at the same point?Perhaps they will say that if the support is sustained by a force at F, then CG will be the goal, and the angle DCG will be the cause of heaviness, but if sustained at G , then FCE will be the cause of heaviness and CF will be the goal.For this it does not appear possible to adduce any but imaginary reasons, because the goal (as they call it) does not appear to have any kind of power that acts sometimes on the side of the larger angle and sometimes on the side of the smaller.But suppose the support to be sustained by two powers at F and G, as it might be through necessity if the power placed at F were so weak that by itself it could sustain only half the weight, and, the power placed at G being equal to that at F, both together could sustain the balance with the weights. Now what angle will be the cause of heaviness?Not FCE, because the support is at CF and is sustained at F; nor DCG, the support being at CG and similarly sustained at G.Therefore the angles will not be the cause of heaviness.Thus the balance DE will not be moved from that position for any such cause.yet they think this opinion is confirmed in two ways.First, they say that Aristotle in his Questions of Mechanics posed these two questions only, and his proofs were based on the greater and lesser angle and on the position of the support of the balance.Next, they declare that experience also bears them out; that is, that the balance DE with its support at CF returns to AB parallel to the horizon, but when the support is at CG the balance moves to FG.But neither Aristotle nor experience favors this opinion of theirs, and indeed quite the contrary is true.They are deceived with regard to experience, since it is clear from experience that this happens when the center of the balance is above or below the balance, rather than when the support is above or below.

[Figure 26]

If the balance AB has its center C above and its support CD below the balance, then when the balance is moved to EF it will return to AB parallel to the horizon.Likewise if the balance has its center C under the balance while its support CD is above, and the balance is moved to EF, it is manifest that the balance will move down on the side of F, the support being above the balance.And in any other position of the support, it will always come out the same.Therefore it is not the support but the center of the balance that is the cause of these effects.

However, it is to be noticed here that it is difficult in fact to make an actual balance (such as we can imagine mentally) that is supported only at one point and has its arms so exactly equidistant from the center, not only as to length but as to breadth and thickness, that all its parts on both sides weigh precisely the same, because matter does not lend itself easily to such exact measurement.Hence if we consider the center to be in the balance, we need not have recourse to the senses, for artificial devices cannot be brought to such a degree of perfection.But in the other things experience may directly teach the appearances.For, although the center of the balance is always a single point, nevertheless, when it is on top of the balance, it does not matter much if the balance is not sustained precisely at that point; because so long as the center remains above, the balance will always behave the same.For a like reason, that which occurs when the center is in the balance never happens when it is below the balance, because there will be a difference if it is not sustained always exactly in that center, and it is a very easy thing for that center to change its position when the balance is moved.

Now, it is certainly true that Aristotle did pose two questions only; that is, why, when the support is above, and the balance is not parallel to the horizon in equilibrium, it returns; whereas, if the support is below, it does not return , and moves farther in the direction of the lower side.But his proofs are not based on the larger or smaller angle and the position of the support, as they pretend, because in this they do not understand the philosopher's meaning when he examines the reason for various effects.And Aristotle is far from attributing these different effects to the angles; rather he says that the cause is the excess, and that [when the support is] above, more of the distance from the perpendicular along one arm of the balance is now on one side and now on the other.

[Figure 27]

Now with the support above at CF, the perpendicular will be FCG, which, according to Aristotle, always points toward the center of the world and which unequally divides the [actual] balance when it is moved to DE, the larger part being on the side of D, which tends downward.Therefore on the side of D the balance will move downward until it returns to AB. But if the support is below at CG, GCF will be the perpendicular, which will likewise divide the balance DE unequally, but the larger part will be on the side of E, so that the balance will move downward on the side of E.Once this is fully understood, one will see that, when the support is above the balance, one must also grant that the center of the balance is on top of the balance, and if below, then the center must be below, as will be manifest farther on.Otherwise Aristotle's demonstration will not prove anything, because, if the center were in the balance, as at C, then the balance might be moved in any way and the perpendicular FG would divide the balance only at the point C and in equal parts.Wherefore the opinion of Aristotle not only does not help them but goes strongly against them.This is clear not only from the second and third propositions of the present book, but also because, if the center is above the balance, the higher weight acquires a greater positional heaviness, considering the return of the balance to the position parallel to the horizon.The contrary happens when the center is below the balance.These things are demonstrated in the following manner, what has been said above being assumed: that is, that the weight will be heavier in that place from which its descent is straighter, and is likewise heavier at the place from which its rise would be straighter.

[Figure 28]

Let the balance AB be parallel to the horizon, with its center C above the balance, and let fall the perpendicular CD.Let the centers of gravity of two equal weights be placed at A and B.Now move the balance to EF.I say that the weight placed at E has greater heaviness than the weight placed at F, and therefore the balance EF will return to AB.First extend the line CD to the center of the world, and let this be S.Then draw the lines AC, CB, EC, CF, and HS, and from the points E and F draw the lines EK and FL parallel to HS.Now since the natural descent of the whole system (that is, of the balance EF in this position together with its weights) is that of its center H along the straight line HS, then likewise the descent of the weights placed at E and F in this position is along the straight lines EK and FL parallel to HS, as we have demonstrated above.Therefore the descent or rise of the weights placed at E and F may be said to be more or less oblique according to the approach or departure [from the vertical] dictated by the lines EK and FL.And since the two sides AD and DC are equal to BD and DC, and the angles at D are right angles, the side AC will be equal to the side CB.Now the point C being fixed, when the points A and B move, they will describe the circumference of a circle whose radius is AC. Hence let the arcs AE and EF be described with the center C, and the points A, B, E, and F will be on the circumference of the circle.But since EF is equal to AB, the arc EAF will be equal to the arc AFB.The common part AF being subtracted, the arc EA will be equal to the arc FB.Now since the mixed angle CEA is equal to the mixed angle CFB, and HFB is greater than CFB, and the angle HEA is less than CEA, the angle HFB will be greater than the angle HEA.When the equal angles HFG and HEK are subtracted, the angle GFB will be greater than the angle KEA; therefore the descent of the weight at E will be less oblique than the rise of the weight at F.And although the descending weight at E and the rising weight at F move through equal arcs, since the descent of the weight from E is straighter than the ascent of the weight from F, the natural power of the weight at E will overcome the resistance to force of the weight F.Hence the weight at E will have greater heaviness than the weight at F, and the weight at E will move downward and the weight at F upward until the balance EF returns to AB; which was to be proved.

The reason given by Aristotle for this effect may here be plainly seen.For take the point N where the lines CS and EF intersect.Since HE is equal to HF, NE will be greater than NF and the line CS (which he calls the perpendicular) will divide the balance EF unequally.So that the part NE of the balance is greater than NF.And since this must be carried downward, the balance EF will move down on the side of E and return to AB.

[Figure 29]

In addition to the things that have been said up to this point, it may be stated that the balance in the position EF will move most swiftly to AB when the line EF, if extended straight, would pass through the center of the world.Let this line be EFS.Since CD and CH are equal, and the circle DHM is described with the center C and radius CD, the points D and H will be on the circumference of the circle.And since CH is perpendicular to EF, EHS will be tangent to the circle DHM at the point H.Then the weight at H (as we have proved above) will be heavier than in any other position on the circle DHM.Therefore the system consisting of the weights E and F together with the balance EF, whose center of gravity is at H, will weigh more in this than in any other position of H on its circle.From this position, therefore, it will move more swiftly than from any other.And if the point H is closer to D, it will weigh less and will move less swiftly from that place, for its descent is always more bent and less straight.Therefore the balance EF will move most swiftly from this place , and the closer it approaches AB the less swiftly it will move.Also the more distant the point H is from the point C, the more swiftly the balance will moves which is manifest not only from what Aristotle says at the beginning of his Questions of Mechanics and from what has been said above, but also from the things that we are going to say in the sixth proposition below.Hence the balance EF will move the more swiftly, the farther it is from its center.

[Figure 30]

Let there be the balance AB with its center below and let there be equal weights at A and B; and move the balance to EF. I say that the weight at F has more heaviness than that at E, and therefore the balance EF will move downward on the side F.Extend the line 1)C on one side to the center of the world S and on the other side to 0, and draw the line HS. From the points E and F draw the lines GEK and FL parallel to each other, and draw CE and CF.Then from the center C with a radius CE draw the circle AEOBF.The points A, B, E, and F are on the circumference of the circle, and it can be shown that the descent of the balance EF together with its weights would be made along the line HS, the descent of the weights at E and F being along the lines CK and FL parallel to HS.And since the angle CFP is equal to angle CEO, the angle HFP will be greater than the angle HEO.But the angle HFL is equal to HEG.Hence if we subtract the equal angles HFP and HEO, the angle LFP will be less than the angle GEO, and the descent of the weight at F will be straighter than the rise of the weight at E.Therefore the natural power of the weight at F will overcome the resistance to force of the weight at E, and thus the weight at F will have greater heaviness than that at E.Hence the weight at F will move down and the weight at E will move up.

[Page 21]

Aristotle's reasoning is equally clear here.For let the point N be the intersection of the lines CO and EF; NF will be greater than NE, and since the perpendicular CO, according to him, divides the balance unequally with the larger part toward F (that is, NF) the balance EF will move downward on the side F since the greater is carried downward.

Similarly from what has been said we deduce that, the farther the balance EF (having its center beneath) is from the position AB, the swifter it will move, because, the farther the center of gravity H is from the point D, the faster will be the motion of the system composed of the weights E and F and the balance EF, until the angle CHS becomes a right angle.And it will also move more swiftly the farther the balance is from the center C.

Besides, we may use their logic and their false assumptions to produce the effects and motions of the balance already explained, so that from this one may see the power of truth and how it forces itself to shine forth even from false things.

[Figure 31]

Assuming the same things, that is, the circle AEBF and the balance AB whose center C is above the balance, move the balance to EF; I say that the weight at E has greater heaviness than the weight at F, and that the balance EF will return to AB.Draw from the points E and F the lines EL and FM perpendicular to AB, which shall be parallel, and let the point N be the intersection of AB and EF.Then, since the angle FNM is equal to the angle ENL and the angles FMN and ELN are, both right angles, and the remaining angles NFM and NEL are also equal, the triangle NLE will be similar to the triangle NMF.Then as NE is to EL, so NF is to FM, and as EN is to NF, EL is to FM.But HE being equal to HF, EN will be greater than NF, and EL greater than FM.Now while the weight placed at E descends along the arc EA, the weight at F rises along the arc FB equal to EA, and the descent of the weight at E partakes (as they say) of the straightness EL, while the rise of the weight at F partakes of the straightness FM, so that the rise of the weight at F partakes less of the straight than the descent of the weight at E.Therefore the weight at E will have greater heaviness than will the weight at F.

Extend the line CD to OP, which shall cut the line EF at the point S.Now since (they say) the farther the weight is from the line of direction OP, the heavier it is, then by this means also they would prove the weight at E to have greater heaviness than the weight at F.Draw from E and F the lines EQ and FR perpendicular to OP.By a like argument it would be shown that the triangle QES is similar to the triangle RFS and that the line EQ is greater than RF, and thus the weight at E will be farther from the line OP than will the weight at F, whence the weight at E will have greater heaviness than the weight at F.From this it appears evident that the balance will return from EF to AB.

[Figure 32]

But if the center of the balance is below the balance, then it will be shown by the same argument that the lower weight should have greater heaviness than the raised weight.Draw from the points E and F the lines EL and FM perpendicular to AB.As before, it is proved that EL is greater than FM, and therefore the descent of the weight at F will partake less of straightness than the rise of the weight at E; hence the resistance to force of the weight at E will overcome the natural inclination of the weight at F, and therefore the weight at E will be heavier than the weight at F.

Extend also CD to O and P and draw from the points E and F the lines EQ and FR perpendicular to that.It will be proved in the same way that the line EQ is greater than FR, and, since the weight placed at E will be farther from the line of direction OP than the weight at F, the weight at E will have greater heaviness than the weight at F.From this it follows that the balance EF moves downward on the side of E.

Thus Aristotle poses only two questions and leaves out the third; that is, the case in which the center of the balance is in the balance itself.But he left this out as a thing well known, as he usually did omit obvious things.Who can doubt that, if the weight is sustained at its center of gravity, it will remain at rest?But perhaps someone will take issue with the things that we have put forth in accordance with his opinion and will say that we have not brought in his entire thought.For in the second part of the second question he asks, "Why, when the support is below, the balance being carried downward and released, it does not rise again, but remains? "Here he affirms not that the balance moves downward, but that it remains, which he seems to have deduced in the last conclusion.But not only does this not bear against us; when properly understood, it greatly assists us.

[Figure 33]

For let there be the balance AB, parallel to the horizon, with its center E under the balance.And since Aristotle considers an actual balance, it is necessary to place the support or something else under the center E; let this be EF, and this will be the support that sustains the center E.Let ECD be the perpendicular .In order that the balance AB may move from this position, says Aristotle, let there be a weight at B which will move the balance downward on the side of B, say, to G, where the obstacle prevents its moving farther downward.Now Aristotle does not say that the balance moves down on the side of B as far as it can, and is left there, as we say; but he would have it that a weight is placed at B, which by its nature will always move downward until the balance rests against its support or something else.When B is at G, the balance will be at GH, in which position it will remain if the weight is taken away; for the side of the balance from the perpendicular toward G (that is, DG) is longer than DH; but the balance does not move farther downward because it will be on the support, or whatever else sustains the center of the balance.And if this did not support it, in his opinion the balance would move downward on the side of G, and DG, being greater, would necessarily be carried farther downward.

But someone might add to this that, if a very small weight were placed at B, it would indeed move the balance downward but not all the way to G, and in this position, according to Aristotle, it should remain if the weight were taken away.This is evident by experience, since the balance tilts more or less when at one end of the balance only there is placed a larger or smaller weight, and this is true enough so long as the center is placed above the balance, but not when it is below or in the balance, as we shall show by way of example.

[Figure 34]

Let there be the balance AB, parallel to the horizon, whose center C is above the balance; let the perpendicular CD be plumb to the horizon, and let this line be extended through D to H.Now, since we consider the weight of the balance, the point D will be the center of gravity of the balance.But if a small weight is placed with its center of gravity at the point B, the center of gravity D of the whole system composed of the balance AB and the weight placed at B will no longer be at D , but it will be in the line DB.Say it is at E, so that DE is to EB as the weight placed at B is to the weight of the balance

AB.Now join C and E; and since the point C is fixed, when the balance moves, the point E will describe the circumference of the circle EFG with radius CE and center C.But since CD is plumb to the horizon, the line CE will not be so.Hence the weight composed of AB and the weight at B will not remain in this position but it will move downward along the circumference EFG according to its center of weight E, until CE becomes plumb to the horizon, that is, until CE gets to CDF.The balance AB will then be moved to KL, in which position the balance together with the weight will remain, nor will it move farther downward.If a heavier weight were placed at B, the center of gravity of the whole system will be closer to B, say at M, and then the balance will move downward until the line joining C and M comes to the line CDH.Hence when a greater or lesser weight is put at B, the balance will be tilted more or less.From this it follows that the weight B will always describe an angle less than a quadrant, since the angle FCE is always acute, nor will the point B ever go all the way to the line CH, because the center of gravity of the weight and the balance together will always be between B and D.The heavier the weight placed at B, the larger will be the arc described, beginning at E and approaching closer to the line CH.

[Figure 35]

But if the balance AB has its center C in the balance, C will also be the center of gravity of the balance; let the line FCG be drawn perpendicular to AB and to the horizon.Then put any weight you please at B, and let the center of gravity now be at E, so that CE is to EB as the weight placed at B is to the weight of the balance.And since CE is not perpendicular to the horizon, the balance AB and the weight at B will not remain in this position but will move downward on the side of B until CE becomes perpendicular to the horizon; that is, until the balance AB comes to FG.Whence it is clear that the weight placed at B always describes a full quadrant.

[Figure 36]

But if the center C is under the balance AB, and DCE is the perpendicular, the placing of a weight at B will similarly make the center of gravity of the system composed of the balance AB and the weight at B be in the line DB, say, at F, in such a way that, as DF is to FB, so is the weight placed at B to the weight of the balance.Draw CF, and, since CD is perpendicular to the horizon, the line CF will not be; hence the system composed of the balance AB and the weight at B will never remain fixed in this position but will move downward if nothing impedes it, until CF comes to DCE, in which position the balance together with the weight will come to rest.Now the point B will be at G and the point A at H, so the balance GH will no longer have its center below, but above.And this will always happen, however small the weight placed at B.Therefore before B goes to G, it will necessarily happen that the balance will strike against the support placed below, or some other thing that sustains the center C, and will stop there.From this it follows that the weight B always moves beyond the line DK and always describes an arc greater than a quadrant, the angle FCE being always obtuse and the angle DCF always acute.And no matter how light the weight placed at B, it will always describe a larger arc [than a quadrant].For the lighter the weight at G, the more the weight at G will rise and the more closely the balance GH will approach a horizontal position.All these things are obvious from what has been said before.

These things proved, it is clear that the center of the balance is the cause of the various acts of the balance, and it is also seen that all the propositions of Archimedes in his book On Plane Equilibrium are true in every position, whether the balance is horizontal or not, provided only that the center of the balance is located within it, and this is the way he considers it.And even if the balance has unequal arms, the same will always happen, and it will be proved in exactly the same way that the center of the balance being situated in different manners will produce different effects.

[Figure 37]

Let there be the balance AB parallel to the horizon, and let there be unequal weights at A and B, so that the center of gravity is at C, and let the balance be hung from this point C.Now move the balance to DE, and it is evident that the balance will rest not only at DE but at any other point.

[Figure 38]

But now let the center of the balance AB be above C at F and let FC be perpendicular to AB and to the horizon.If the balance shall be moved to DE, the line CF will move to FG, and since this is not perpendicular to the horizon, the balance DE will move down on the side of D until FG returns to FC, when the balance DE will be at AB, in which position it will rest.

[Figure 39]

But if the center F of the balance shall be below the balance, and the balance is moved to DE, it is evident in the first place that the balance will rest at AB and that at DE it would move downward on the side of E, since the line FG is not perpendicular to the horizon.

[Figure 40]

From these things just finished, if the balance were curved or if the arms of the balance formed an angle, and the center were variously placed-although strictly speaking this would not be a balance-we might nevertheless demonstrate various effects in it also.Thus let the balance be ACB, which turns about the center C, and draw the line AB so that the curve or angle A CB is above the line AB, and place the centers of gravity of the weights at A and B, which will rest in this position.Next move the balance from this position as to ECF.I say that the balance ECF will return to ACB.

[Figure 41]

Find the center of gravity of the whole system, D, and join C to D.Now since the weights A and B are at rest, the line CD will be perpendicular to the horizon.Therefore when the balance is at ECF, the line CD will be at CG, and, since this is not perpendicular to the horizon, the balance ECF will return to ACB.The same will happen if the center C is placed above the balance as at H.

[Figure 42]

Now if the curve or angle ACB shall be beneath the line AB, we may show in the same way that the balance ECF, whether its center is at C or H, must move downward on the side of F.

And if the angle ACB is above the line AB at the center of the balance H, and if the line CH sustains the balance, when the balance is moved to EKF, it will return to ACB.

[Figure 43]

But if the center of the balance is D, one may move the balance in any way, and wherever it is placed, it will rest.

[Figure 44]

Then if the point H shall be beneath the line AB, the balance EKF will move downward on the side of F.

[Figure 45]

And for similar reasons if the angle ACB shall be beneath the line AB, and the center of the balance is at H, and the balance is sustained by the line CH, then, if the balance is moved from this position, it will move downward on the side of the lower weight.And if the center of the balance is at D, it will rest wherever it is left.And if the center is at K and the balance is moved from that position, it will return in any event to the same place.All which things are manifest from what we said at the beginning.Similarly if the center of the balance is placed in one of the arms of the balance, either within or without, or any other way, we shall find the same things.

Comment by Pigafetta

At this point it is well to notice what might have been said toward the end of page 26v, where the author wrote: "In addition to this we may consider the things that may be deduced in the same way." Now our author is the first to have considered the balance in detail and to have understood its nature and its true quality.For he is the first of all to have shown clearly the way of dealing with it and teaching about it, by propounding three centers to be considered in its theory : one is the center of the world, another the center of the balance, and finally the center of gravity of the balance: for in this was a hidden secret of nature.Without these three centers, it is clear that one could not come to a perfect knowledge or demonstrate the various properties of the balance, which were hidden in the variety of arrangements of the center of the balance-that is, whether the center of the balance is above its center of gravity, or below, or in the very center of the balance.These the author shows in the three preceding demonstrations, that is, in the second, third, and fourth propositions.In the second, he shows when the balance returns to a horizontal position, in the third when it not only does not return but moves still farther, and in the fourth that, when a balance is sustained at its center of gravity, it remains at rest wherever it is left.This last effect in particular has not been dealt with before, or seen, or even suggested by anybody besides this author.Indeed, until the present time it has been held to be false and impossible by all our predecessors, who not only have given many arguments attempting to prove the contrary, but have even assumed it to be certain that experience shows the balance never to remain fixed except when parallel to the horizon.This is quite contrary to reason, first because the demonstration of the above fourth proposition is so clear, easy, and true that I do not know how it could be contradicted in any way; and second, their view is contrary to experience, inasmuch as our author has very cleverly fashioned precise balances for the purpose of showing this truth, one of which I saw in the hands of the illustrious Giovanni Vicenzo Pinelli 28 given to him by the author himself, and, because it was sustained at its center of gravity, it could be moved to any position and would rest at any place where it was left.True it is that in performing this experiment one might not act hastily, for it is an extremely difficult thing (as the author says above) to make a balance which is sustained precisely at the center of its arms and at its precise center of gravity.For this reason it is good to remember that, when anyone tries to perform such an experiment and does not succeed, he should not be discouraged, but rather should say that he had not been careful enough, and should try repeatedly until the balance is just and equal and is sustained precisely at its center of gravity.

And though others have touched on the other two propositions (that is, when the balance almost returns to a horizontal position, and when it moves in a contrary way), yet the truth of this has never been understood except by this author, and others have not gone far enough to have made a distinct consideration of the center by the balance in three ways, as I have explained.To the extent that they have done anything about this matter, they have done it confusedly and with poor demonstrations from which no one can draw clear and firm conclusions.These predecessors of ours are to be understood as being the modern writers on this subject cited in various places by the author, among them Jordanus, who wrote on weights and was highly regarded and to this day has been much followed in his teachings.Now our author has tried in every way to travel the road of the good ancient Greeks, masters of the sciences, and in particular that of Archimedes of Syracuse (the most famous prince of mathematicians) and Pappus of Alexandria, reading them, as he tells us, in their own language and not translated, because for the most part they are so unsatisfactorily treated that it is very hard to draw any profit from them whatever.

And to the end that this new opinion of his, fully demonstrated in the aforesaid fourth proposition, should be completely clear, he has not been content to demonstrate it with vivid and certain reasoning alone, but, like a good philosopher; proceeding by the path of true doctrine and well-founded science (imitating Aristotle, who at the beginning of his books, in quest of the best doctrine, has given the contrary opinions of the ancients, analyzing the reasons which they accepted), he has wished, because there is but one truth, to propound the opinions of his predecessors and examine the reasons by which they seem to prove the contrary, and to resolve these, showing their fallacy in the present argument, which commenced on page 5v and ends at this point.Which argument will serve to clear up what is usually said to be the opinion of the ancients.And since it contains things of the highest theoretical value, especially with regard to the consideration of the place at which a single weight placed on the arm of the balance is heaviest, in order to understand it one must read it and study it with great diligence.

And certainly our author has been not only the first to discover this truth, but also the first to show in what manner one must consider and theorize concerning the whole subject; and with his theory he proves again and confirms the various effects and events of the balance already demonstrated in the adjoining three propositions, showing also how until now these things have been badly considered by others, and on false principles.Moreover, as a confirmation of the truth, he adds that they did not know how to construct their proofs; for by their own mode of theorizing and their very own reasons, he proves his opinions to be most true, supporting them always on the doctrine of Aristotle and making it clear that he is in accord with him in questions of mechanics.In dealing with this matter, the author also raises some new questions, very beautiful and curious, and then solves them clearly.Finally, in order that nothing might be lacking to the complete knowledge of this subject, he has dealt with balances whose arms are unequal and those which have curved and bent arms.In a word, it may well be said that in this argument everything is included that can be determined concerning this subject.The theories are beautiful and very subtle and are to be looked at and considered with much attention by anyone who delights in and attends to these noble and necessary studies.

Wherever the Latin word equilibrium is read, it means "equally counterpoised," that is, weighing as much on one side as on the other in equal scales or balances.Librar con giuste lance, Petrarch said.

PROPOSITION V [Page 26]

[Figure 46]

If two weights are attached to a balance and the balance is divided between them in such a way that the parts correspond inversely to the weights, they will weigh as much at the points where they are attached as if each were suspended from the point of that division.

PROPOSITION VI

[Figure 47]

Equal weights suspended from a balance have heaviness in proportion to the distances at which they are suspended.

Corollary.-From this it is evident that, the farther the weight is from the center of the balance, the heavier it is, and consequently the more swiftly it will move.From this one may, furthermore, easily demonstrate the theory of the steelyard
.

Comment by Pigafetta

Corollary is a Latin word employed by all Italian writers on this subject, nor did it displease Dante in the Twenty-eighth Canto of his Purgatory."Also called a corollary, for example," as Varro says in his first book on the Latin language, "is anything over and above that which would normally be paid in buying something."In ancient times when the actors in tragedies, comedies, and other poems carried off their scenes well and pleased their audiences, something was given to them in addition to the fixed price-a corollary for each-one-that is, a small crown to be placed on their foreheads and added to their rewards.Thus in the mathematical sciences it is customary to add certain things to propositions, as belonging to them and consequences of them, which take their rise from the things previously demonstrated, and correspond to them; and these are not propositions or problems or lemmas, but, as indicated before, are called corollaries many of which are given with their proofs.

The steelyard may also be used in another way to make the weight of things known.

PROPOSITION VII

[Figure 48]

Problem: Given an indefinite number of weights on the balance, suspended at any places, to find a center on the balance from which, if the balance were to be hung, the given weights would be in equilibrium.

Comment by Pigafetta

Under the name of "proposition" is included "problem," also a Greek word; the problem goes beyond the proposition in that it proposes and shows how to achieve some result, whereas the proposition gives the bare theory only; and this is the difference between a proposition and a problem.

[Figure 49]

Let there be the balance AB and let there be given any number of weights C, D, E, F, and G hanging from the balance at the points A, H, K, L, and B.It is required to find the center of the balance from which, if [the balance were] suspended, the weights would be at rest.

[Figure 50]

Divide AH at M so that HM is to MA as the weight C is to the weight D; then divide BL at N so that LN is to NB as the weight G is to the weight F, and divide NM at O so that MO is to N as the weights F and G are to the weights C and D finally, divide KO at P so that KP is to PO as the weights C D, F, and G are to the weight E.Now since the weights C, D, F, and G weigh as much at O as C and D at M and F and G at N, the weights C and D at M, F and G at N, and E at K will balance if they are suspended from the point P.Inasmuch 3 as the weights C and D weigh as much at M as at A and H, and if F and C at N as much as at L and B, the weights C, D, F, and G hanging from the points A, H, L, and B and the weight E from K, if the balance is suspended from P, will weigh equally and will be at rest.Therefore P is the center of the balance from which the given weights will be at rest, which was to be achieved.

Corollary.-From this it is clear that if the centers of gravity of the weights C, D, E, F, and G were at the points A, I1, K, L, and B, the point P would be the center of gravity of the whole system composed of these weights.This is obvious from the definition of the center of gravity, inasmuch as the weights will be at rest if they are sustained from the point P.

End of the Balance

On the Lever
LEMMA

[Page 29]

Let there be four magnitudes A, B, C, and D, and let A be greater than B and C be greater than D.I say that the ratio of A to D is greater than that of B to C.

Since the ratio of A to C is greater than that of B to C and the ratio of A to D is greater than that of A to C, the ratio of A to D will be greater than that of B to C.Q. E. D.

PROPOSITION I

The power that sustains a weight attached to the lever has the same proportion to that weight as the distance along the lever between the fulcrum and the point of suspension has to the distance between the fulcrum and the power.

From this it can easily be demonstrated that the closer the fulcrum is to the weight, the smaller the power required to sustain the weight.

[Figure 51]

Corollary.-Whence it can be quickly deduced that, AF being less than FB, a smaller power is required at B to sustain the weight D; and if they are equal, it is equal; and if AF is greater, the required power is greater.

The lever may be used in a second mode.

[Figure 52]

Proof: Let there be the lever AB with its fulcrum B, and let the weight C be attached at D between A and B, and let the power at A sustain the weight C.I say, that as BD is to BA, so is the power at A to the weight C---.

[Figure 53]

Corollary I.-From this also, as before, it may be shown that, if the weight E is placed closer to the fulcrum B, as at H, a smaller power is required at A to sustain the weight.

Corollary II.-It also follows that the power at A is always less than the weight E.

[Figure 54]

Corollary III.-From this likewise it may be deduced that, if there are two powers, one at A and the other at B, and both sustain the weight E, the power at A will be to the power at B as BC is to CA.

[Page 31]

Corollary IV.-I t is furthermore evident that the two powers at A and B taken together are equal to the weight E.

PROPOSITION III

We may also use the lever in a third mode.

[Figure 55]

Let there be the lever AB with its fulcrum at B, and let the weight C be hung from the point A, and let it be the power at D, somewhere between A and B, that sustains the weight C.I say that, as AB is to BD, so is the power at D to the weight C---.

Corollary I.-From this it is also clear, as before, that if the weight is closer to the fulcrum B, as at H, the weight must be sustained by a smaller force.

Corollary II.-It is likewise evident that the power at D is always greater than the weight C.

PROPOSITION IV

[Page 32]

If the power shall move the weight hung from the lever, the space through which the power moves will be to the space through which the weight is moved as the distance from the fulcrum to the power is to the distance from the fulcrum to the point from which the weight is hung.

[Figure 56]

Let there be the lever AB with its fulcrum C, and let the weight D be attached at the point B, and let the power at A move the weight D by means of a leverAB.Then the space of the power at A is to the space of the weight as CA is to CB---.

But let there be the lever at AB, whose fulcrum is B, and the moving power is at A and the weight at C; I say that the space of the moved power to the space of the weight carried is as BA to BC---.

Corollary.-From these things it is evident that the ratio of the space of the power which moves to the space of the weight moved is greater than that of the weight to the same power.

[Figure 57]

For the space of the power has the same ratio to the space of the weight as that of the weight to the power which sustains the same weight.But the power that sustains is less than the power that moves; therefore the weight will have a lesser ratio to the power that moves it than to the power that sustains it.Therefore the ratio of the space of the power that moves to the space of the weight will be greater than that of the weight to the power.

PROPOSITION V

[Figure 58]

The power that sustains the weight in any way by means of the lever will have the same proportion to the weight as that of the distance from the fulcrum to the point on the lever, vertical to the center of gravity of the weight, to the distance between fulcrum and the power.

PROPOSITION VI

[Figure 59]

Let there be the straight line AB, and perpendicular to it the line AD, prolonged on the side of D to C.Join C and B and extend this to E.Then let there be drawn from the point B other lines between AB and BE, for example, BF and BC, equal to AB.From the points F and G let there be drawn the lines FH and CK perpendicular to the above lines, and let these be equal to one another and to AD, as if BA and AD were moved to BF and FH and to BC and GH.Draw CH and CK, which cut the lines BF and BG at the points M and N.I say that BN is shorter than BM, and BM than BA.

[Figure 60]

If the equal triangles BFH and BCK are between BC and BA below, and if there are added the lines HC and KC which cut the lines BF and BG extended at the points M and N, then BN will be greater than BM and BM than BA .

PROPOSITION VII

Let there be the line AB and the perpendicular AD extended as far as C.Draw CB and extend this to E. Between AB and BE draw BF and BC equal to AB, and from the points F and C draw the lines FH and CK also equal to AD and perpendicular to BF and BC, as if BA and AD were moved to BF and FH, or to BG and GK.Draw CH and CK, which cut the lines BF and BG at the points M and N.I say that BN is greater than BM and BM than BA.

And if the triangles BFH and BCK are placed between AB and BC below, and the lines CHO and CKP are drawn, which cut the lines BF and BC at the points M and N, then BN will be less than BM and BM than BA.[Cf. second statement and diagram, Prop. VI above].

PROPOSITION VIII

If the power sustaining the weight which has its center of gravity on a horizontal lever is given, then, the more the weight is raised from this position by means of the lever, the smaller the power required to sustain it.But if it shall be lower, the power is greater.

[Figure 61]

From this it is easily deduced that the power at A is to the power at E as CL is to CM.

In addition to this, if there is another power at B, so that there are two powers that sustain the weight, the power at B, which sustains the weight PQ by means of the lever BO, will be less than the weight CD on the lever BA.On the other hand, a greater power is required at B to sustain the weight FC, by means of the lever BE, than the weight CD on the lever AB.

[Figure 62]

Corollary .-From these things it is evident that, if a power raises, by means of a lever, a weight whose center of gravity is above the lever, then the more the weight is raised, the smaller becomes the power necessary to move the weight .

PROPOSITION IX

[Figure 63]

If the power sustaining a weight that has its center of gravity under the lever is given when that [lever] is horizontal, then the more the weight is raised from this position by means of the lever, the more power will be required to sustain it; but if it is lowered, the power becomes less.

PROPOSITION X

[Figure 64]

The power sustaining a weight that has its center of gravity in the lever itself will always be the same no matter how the weight is moved by means of a lever.

PROPOSITION XI

If the ratio of the distance along the lever between fulcrum and power to the distance between fulcrum and that point on the lever vertical to the center of gravity of the weight is greater than the ratio of the weight to the power, the weight will be moved by the power.

PROPOSITION XII

Problem: To move a given weight by means of a given lever with a given power.

[Figure 65]

Let the weight A be 100 and the power that must move it be 10, and let the given lever be BC.It is required that the power of 10 shall move the weight of 100 by means of the lever BC.Divide BC at D in such a way that CD has the same ratio to DB that 100 has to 10, i. e., 10 to 1 ---.Take between B and D any point you wish, such as E, and make E the fulcrum---.

Corollary.-From this it is manifest that, if the given power is greater than the given weight, the weight can be moved whether the lever has its fulcrum between the weight and the power, or has the weight between the fulcrum and the power, or finally if the power is placed between the weight and the fulcrum.But if the given power shall be less than or equal to the given weight, it is likewise clear that the weight can be moved only if the lever is such that the fulcrum is between the weight and the power, or the weight is between the fulcrum and the power.

PROPOSITION XIII

Problem: Given an arbitrary number of weights suspended from arbitrary points of a lever whose fulcrum is also given, to find a power which will sustain these weights at a given point.

[Figure 66]

Let there be given the weights A, B, and C on the lever DE (with its fulcrum at F), suspended from the points D, C, and H, and a point E at which the power must be applied---.Divide DC at K in such a way that DK is to KC as the weight B is to A; then divide KH at L so that KL is to LH as the weight C is to the weights B and A.As FE is to FL, make the sum of weights A, B, and C be to the power which must be placed at E---.

PROPOSITION XIV

Problem: To make a given power move an arbitrary number of weights at arbitrary places on a given lever.

[Figure 67]

Let the given lever be DE, let the given weights be placed as above, and let A be 100, B, 50, and C, 30; and let the given power be 30.Find the point L as before; then divide LE at F in such a way that FE is to FL as 180 is to 30 (that is, as six is to one), and if F is the fulcrum, the power of 30 at E will sustain the weights A, B,- and C.Therefore between L and F take some point such as M, and make this the fulcrum---.

PROPOSITION XV

[Page 38]

Problem: But since in moving weights with a lever, the lever also has weight, which has not been mentioned up to this point, we shall demonstrate how to find the power which will sustain the lever in a given point, the fulcrum being likewise given.

[Figure 68]

Let there be the lever BA with its fulcrum at C, and let there be the point D at which the power must be applied which must sustain the lever AB so that it remains at rest.From the point C draw the vertical line CE, dividing the lever AB into AE and EF.Let C be the center of gravity of AE and H the center of gravity of EF, and draw the vertical lines CK and HL cutting the line AF at the points K and L.Now since the lever AB is divided into two parts, that is, AE and EF, the lever AB is itself two weights placed in the balance AF, whose support is C.Hence the weights AE and EF are placed as if they were suspended at K and L.Divide KL at M in such a way that KM is to ML as the weight of the part EF is to the weight of the part AE; and in the proportion of CA to CM make the weight of the whole lever AB be to the power which, if applied at D (provided that DA is vertical), would counterpoise the lever; that is, would sustain the lever AB by its pressure.

Next, a weight hung from the lever is to be added, such as the weight P hung from A, and the power is to be applied at B in such a way that it sustains the lever AB together with the weight P.

[Figure 69]

Divide AM at Q in such a way that AQ is to QM as the weight of the lever AB is to the weight P.In whatever ratio CF is to CQ, make the combined weight of AB and P be to the power placed at B.It is clear that the power at B would sustain the lever AB together with the weight P.For if CA were to CM as AB is to P, the point C would be their center of gravity , and hence the lever AB together with the weight P would remain at rest without the power placed at B.Now if the center of gravity of the weights is between C and F, as at O, then, as CF is to CO, so AB and P together are to the power which, when applied at B, will sustain the lever AB with the weight P.The same may be done when there are more weights along the lever AB, no matter where and in what way they are arranged.In addition to these things, one may see (as we have shown in the fourteenth proposition) the way in which we may move the given weights, placed anywhere on the lever, by means of a given power with a given lever.This we can do by considering not only the weight of the lever itself but also the other properties which have been demonstrated above independently of the weight of the lever.All may be shown by consideration of the combined weight of the lever and its weights or [the lever] without additional weights.

End of the Lever

On the Pulley

By means of the pulley things may be moved in many ways, but, since the theory is the same for all and in order to present the thing most clearly, it is to be understood in that which is about to be said that the weight is always to be moved upward at right angles to the horizontal plane.

Let there be the weight A which is to be raised vertically in the usual way.Held from above is the block containing two pulleys with their axles at B and C.Let another block be attached to the weight, also with two pulleys at D and E; and around all the pulleys lead the rope tied at one end, say, at F.Apply the power at C, so that, when this descends, A is raised, as Pappus shows in the eighth book of his Collections, Vitruvius in the tenth book of his Architecture, and others.

[Figure 70]

Now let us show how the pulley may be reduced to the lever, why a great weight is moved by a small force, in what way, in what time, why the rope must be secured at one end, what is the function of the pulley that is placed below and what that of the one above, and how one may find any given proportion between the power and the weight .

PROPOSITION I

If the rope is led around the pulley that is fastened from above, and one end of the rope is tied to the weight while the power that sustains the said weight is applied to the other end, then the power will be equal to the weight.

[Figure 71]

Corollary.-From this it is evident that the same weight can always be sustained by the same power without any assistance from this pulley .

PROPOSITION II

[Page 40]

If the rope is led around a pulley to which the weight is directly attached, and one of its ends is fastened to some place while the power that sustains the weight is applied to the other end, then the power will be one-half the weight.

... Since the power sustains the pulley by means of the rope, and the pulley sustains the attachment to which the weight is fixed, the whole weight will be at the center E.Now if we assume the power applied at C to act at D (because it is entirely the same thing), BD will be a lever with its support at B and its weight attached at E and the power applied at D; the rope BF being motionless, B serves as the fulcrum ---.Now since the power has the same proportion to the weight that BE has to BD --- the power at C will be one-half the weight A.

[Figure 72]

Corollary I.- From this it is evident that the weight is sustained in this way by a power of only one-half that which would be required without the aid of such a pulley.

Corollary II.-It is also evident that, if there were two powers which sustain the weight A, one at C and the other at F, then the two together would be equal to the weight A, and each of them would sustain one-half of the weight A.

Corollary III.-It is likewise evident why the rope must be secured at one end.

PROPOSITION III

[Page 41]

If two pulleys are given, one attached above and the other below, to the latter of which the weight is attached, and a rope is given, led around both with one of its ends fastened at some place, then the power which sustains the weight applied to the other end will be equal to one-half the weight.

[Figure 73]

Corollary.-If there are two powers at N and L, they will be equal.For each will be equal to one-half of A.

PROPOSITION IV

[Figure 74]

Let there be a lever AB with its fulcrum at A, divided into two equal parts at D; let the weight C be suspended from D, and let there be two equal powers applied at B and D which are to sustain the weight C.I say that each of these powers is one-third of the weight C .

And if two levers AB and EF, bisected at C and D, have their fulcrums at A and F, while the weight C is equally supported by both levers, and if there are two equal powers at B and C, it may be shown that each of those powers is one-third of the weight C.

PROPOSITION V

[Page 42]

[Figure 75]

If two pulleys are given, one supported from above and the other attached to the weight, with a rope led around them, one end of which is attached to the lower pulley, then a power equal to one-third of the weight applied to the other end will sustain the weight .

... The rope segment MD supports as much as HB, which in turn supports as much as FL; and since the rope MD will sustain as much as FL, it is as though equal weights were applied at D and L.Inasmuch as equal weights are sustained by equal powers, the powers at M and L will be equal, as if they were applied at D and E.Therefore, since the weight A is centrally attached to the lever BD and the two powers placed at D and E are equal and sustain the weight, then B will be the fulcrum, and each power, whether applied at D and E or M and L, will be one-third the weight A---.

[Figure 76]

Corollary.-From this it is evident that each of the rope segments MD, FL, and HB sustains one-third of the weight A.

... And in order that we need not return to say the same thing again, it should be noticed that the power at O is always equal to that at M; that is to say, if the power at M is one-fourth or one-fifth (or some such part) of the weight, then the power at O will be one-fourth or one-fifth of the weight, etc.

Comment by Pigafetta

Now some people might question these demonstrations about pulleys, for instance this fifth proposition which I select as the best example, asking whether in fact experiment is in agreement with theory as to the ratios of forces and weights.For in mathematical demonstrations, all lines are assumed to be without breadth or thickness, and all things are abstracted from actual matter, so that it is easy to persuade ourselves of the mathematical truth.But experience very often shows something different, and we find ourselves deceived, for actual matter changes things quite a bit.In this proposition it is shown reasonably that, for two pulleys and one rope, the force will be one-third of the weight---.Somebody might consider this very dubious, because the pulleys and their attachments, the ropes, and so on offer resistance to the force, and also have weight of their own, so that the [calculated] force may not be able to sustain the weight.We reply that these things may well offer resistance to the moving of the weight, but not to the sustaining of it; and it is necessary to note carefully that the author in these demonstrations speaks only of forces sustaining the weights so that they do not fall down; not about moving them--- Thus we have to do only with a weight at rest, and will take into consideration only its counterweight, which is the function of the sustaining power.Hence neither in the pulleys nor anywhere else is there any resistance, and the theoretical proof will always come out very well; indeed, experience shows that the more resistance there is, the more easily the force sustains the weight---.But to know how much force must be added to the power in order that it may sustain the whole weight, including the lower pulley and the ropes, take the lower pulley and part of the rope as an additional weight---.

PROPOSITION VI

[Figure 77]

Let there be two levers AB and CD, bisected at E and F, with their fulcrums at B and D; and let there be the weight G, suspended in such a way that it weighs equally on E and F, and two equal powers at A and C which sustain the weight.I say that each of the powers is one-fourth of the weight G.

[Figure 78]

But if there shall be three levers AB, CD and EF, bisected at C, H, and K, with their fulcrums B, D, and F, and if the weight is similarly suspended from G, H, and K, while three equal powers A, C, and E sustain the weight, it will be likewise seen that each is one-sixth of the weight L.And in the same manner, if there were four levers and four weights, each power would be one-eighth of the weight; and so on.

PROPOSITION VII

If three pulleys are given, one of which is suspended from above and two from below, and to these latter a weight is attached, and rope is wound around them and one end secured, then a power applied at the other end equal to one-fourth of the weight will sustain the weight.

Corollary I.-From this it is evident that each of the ropes EF, GK, LN, and OP sustains one-fourth of the weight A.

[Figure 79]

Corollary II.-It is also clear that the pulley whose center is C sustains no less than that whose center is B .

PROPOSITION VIII

Let there be two levers AB and CD, bisected at E and F, with their fulcrums at A and C; and let the weight G be suspended in such a way that it weighs equally on E and F; and let there be three equal powers at B, D, and E which sustain the weight G.I say that each of these powers alone is one-fifth of the weight G.

PROPOSITION IX

[Figure 80]

If there are four pulleys, one of which is attached from above and one of which is attached to the weight, and the rope is led around with one of its ends tied to the lower pulley, while the force is applied to the other end, the force that sustains the weight will be one-fifth of the weight .

Comment by Pigafetta

In this treatise on the-pulley (as in the others), the author assumes that all readers of his book on mechanics understand arithmetic and geometry; he has therefore always adhered to that precise and demonstrative style which is customary among good mathematicians, using the special words of the science, some of which I have been able to popularize so that anybody can easily understand them---.But for certain Latin terms used for special proportions, we have no equivalent in our language, and I have been obliged to leave them untranslated for lack of ordinary words to express them---.

Corollary.-From this it is evident that the [lower] pulleys to which the weight is attached make it possible for the weight to be sustained by a lesser power than itself-something which is not accomplished by the upper pulleys.

PROPOSITION X

Let the rope be led around a pulley suspended from above, and let the weight be attached to one end, while the power that moves it is applied to the other.Then the said power will always move the weight as by a lever always parallel to the horizon.

[Figure 81]

Under the above assumption, the space of the power that moves the weight is equal to the space of the weight that is moved .

Moreover, the power moves the weight through an equal space in an equal time, whether the rope is wound around a pulley supported from above, or [lifts the weight] without any pulley at all, provided that the movements of this power are equal in speed.

PROPOSITION XI

[Figure 82]

Let the rope be led around a pulley to which the weight is attached, and let one end of the rope be sustained at some place while the power that moves the weight is applied at the other.Then the power will always move the weight as with a lever parallel to the horizon .

In these circumstances the space of the power that moves the weight is double the space of the weight that is moved.

The power will move the weight in an equal time through half the space --- which would be traversed without the pulley, provided that the speeds of the power are equal.

PROPOSITION XII

[Figure 83]

If the rope is wound around many pulleys, and one end is fastened at some place, and the force that moves the weight is applied to the other end, the force will always move the weight as by a lever parallel to the horizon.

PROPOSITION XIII

If the rope is wound around a pulley supported from above and another to which the weight is attached, and one end of the rope is tied to the lower pulley while the power that moves the weight is applied to the other end, the space passed through by the power is three times that of the space of the weight moved.

PROPOSITION XIV

[Figure 84]

If the rope is led around three pulleys in two blocks, with a single pulley attached from above and two below attached to the weight, and one end of the rope is secured at some place while the power that moves the weight is applied to the other, then the space traversed by the power will be four times that through which the weight moves.

Corollary I.From these things we see why the ratio of the weight to the power that sustains it is the same as the ratio between the space of the moving power and the space of the weight moved.

[Figure 85]

Corollary II.It is likewise evident from what has been said that the pulleys to which the weight is attached have the function of reducing the space passed through by the weight with respect to the power that moves it, and that the weight goes through the same space in a longer time than [it would if moved] without the pulleys; which function does not belong at at all to the pulley attached from above .

PROPOSITION XV

If the rope goes around the pulley of a block held from above by the power, and one end of the rope is fastened at some place while the weight is attached to the other, then the power will be twice the weight.

PROPOSITION XVI

Under the same assumptions, if the power that moves the weight shall be at H, it will move as if it were a lever parallel to the horizon.

Under these assumptions the space of the weight moved is twice that of the space of the power that moves.

[Figure 86]

Corollary.From this it is evident that a given weight is drawn with this sytem of pulleys by the same power in an equal time through twice the space traversed without pulleys, provided that the movements of the power are equal in speed .

PROPOSITION XVII

[Figure 87]

If an upper pulley is suspended from above by the power and a lower pulley is securely fixed, while the rope is wound around them with one of its ends tied to the pulley above and the other attached to the weight, then the power will be three times the weight .

PROPOSITION XVIII

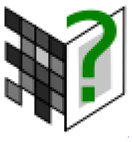
If there are two blocks of two pulleys each, one of which is held from above by the power while the other is secured below , and a rope is wound around them with one of its ends attached elsewhere than to the upper block and the other end holds the weight, then the power will be four times the weight.

Corollary.ÑFrom this it is evident that if the rope were fastened at C and wound around the pulleys whose centers are B, C, and D, then the power at R would sustain four times the weight Q; for the pulley whose center is at A does nothing---

Corollary.ÑIn these things it is evident that the pulleys of the upper block constitute the reason for which the weight is moved by a greater power than itself, through a greater space than that of the power or through equal space in less time; but this is in no way caused by the lower pulleys.

PROPOSITION XIX

If there are two pulleys, one of which is held from above and the other is held by the power [O] that sustains [weight M], while the rope around them has one of its ends secured and the other attached to the weight, then the power will be twice the weight.



[Figure 88]

Corollary.ÑFrom this it is evident that the pulley below in this case causes the weight to be moved by a greater power than itself and through a greater space than that of the power (or through equal space in less time), which cause does not belong to the pulley attached from above .

PROPOSITION XX

If there are two pulleys, the upper one of which is sustained by the power and that below is attached to the weight, while the rope is secured at one end and the other end is attached to the pulley below, the weight will be one and one-half times the power.

PROPOSITION XXI

[Page 51]

[Figure 89]

If there are three pulleys, one of which is sustained from above by the power while the other two are below and attached to the weight, and one end of the rope is fastened while the other is attached to the upper pulley, the weight will be one and one-third times the power .

PROPOSITION XXII

[Figure 90]

If there are two pulleys, of which one is sustained from above by the power and the other is attached to the weight, while one end of the rope is fastened and the other is attached to the upper pulley, then the power will be one and one-half times the weight .

PROPOSITION XXIII

If there are two pulleys, one of which is sustained by the power from above while the other is attached to the weight below, and the rope has both ends fastened elsewhere than to the pulleys, then the power will be equal to the weight.

[Propositions 24 and 25 deal similarly with variant systems].

PROPOSITION XXVI

Problem: To find [systems of pulleys such that] the ratio of the weight to the power that sustains the weight shall be as five to three.

[The author also gives analogous demonstrations for various other problems].

[Figure 91]

Corollary.ÑFrom these things it is evident that the space of the power that moves has always a greater ratio to the space of the weight moved than that of the weight to the same power.This is evident from what was said in the corollary to the fourth proposition concerning the lever .

PROPOSITION XXVII

Problem: To move a given weight with a given power by means of pulleys.

The power is greater than the weight, or equal to it, or less than the weight.If the power is greater, the given weight may be moved without any instrument, or a rope around a pulley supported from above will move it ---

But if equal, it will move the weight by a rope around a pulley attached thereto, because the power that would sustain the weight is one-half the weight---If it is less, suppose the weight is 60 and the power is 13.Find the power at A that will sustain the weight, which [power] is one-fifth the weight; so the power at A that sustains the weight is 12 --- so a power of 13 at A will move the weight.

It is also to be noted that, in the moving of weights, sometimes it is more convenient to move the power downward rather than upward, so the rope may be carried over the pulley whose center is C ---

PROPOSITION XXVIII

[Figure 92]

Problem: To provide that the power moving the weight and the weight itself shall move through given spaces which are commensurable.

--- We can achieve the result with a single rope by what was said in the twenty-second and twenty-fifth propositions.And if we wish to use more ropes, we may do it in an infinite number of ways---

Corollary I.ÑFrom these things it is manifest that in an infinite number of ways, by means of pulleys, we can achieve any given proportions between the weight, the power, and the spaces through which they are moved.

Corollary II.ÑIt is also evident that the more easily the weight is [to be] moved, the greater will be the time [required]; and the greater the difficulty with which the weight is moved, the shorter the time; and conversely.

End of the Pulley

On the Wheel and Axle

[Figure 93]

The construction and nature of this instrument is explained a by Pappus in the eighth book of his Mathematical Collections.AB is called the axle, CD the drum on the same center (which we will call the wheel), and those rods that are inserted in the holes of the wheel and designated EF, GH, and so on, we shall call handles.The power or the force is always applied to the handles, as at F, and this turns the wheel, which in turn moves the axle, which draws up the weight K suspended by the rope LM around the axle.It now remains for us to show how weights may be moved by a small force with this instrument, and in what manner, and, moreover, to show the rule of the times and spaces of the moving power and of the weight moved; and finally, to reduce this instrument to the lever.

PROPOSITION I

[Page 55]

The power sustaining the weight by means of the wheel and axle is in the same ratio to the weight as the radius of the axle to the radius of the wheel including the handle.

[Figure 94]

Corollary.ÑIt is evident that the power is always less than the weight.

--- It should be noted that, if the weight were applied to a different handle, say T, and should sustain the weight K so that the weight applied at T and the weight K suspended from around the axle should remain motionless, then the weight at T must be heavier than the weight M applied at F---

If in place of the weight at T one were to apply a living force to sustain the weight K, acting as if it wished to reach the center of the world, as did the weight applied at T by its own nature, then this power will be equal to the weight at T --- but if each power able separately to sustain the weight, at T as well as at F around the circumference THFN, were to move as if the handle were pressed by the hand, then the same power placed at F or at T would be able to sustain the same weight K---

Now the power moves the weight by means of the lever FB; that is, when the power at F rotates the wheel, the axle also rotates, and FB serves the function of a lever with its fulcrum at C, its moving power at F, and the weight applied at B---

--- Therefore let the power be where it will, the space of the power will be to the space of the weight moved as CF is to CB; that is, as the radius of the wheel to the radius of the axle.

Corollary I.ÑIt is evident from the above that, as the weight is to the power sustaining the weight, so is the space of the moving power to that of the weight moved.

Corollary II.ÑIt is also evident that the space of the moving power has always a greater ratio to the space of the weight moved than that of the weight to the power.

Corollary [III].ÑIt is further evident that the more easily theiweight is moved, the longer time it will take [to raise the weight a given distance], and conversely.

PROPOSITION II

Problem: To move a given weight by means of the wheel and axle, with a given power.

[Figure 95]

Let there be given a weight of 60 and a power of 10.Draw the straight line AB, divided at C in such a way that AC is to CB as 60 is to 10.And if CB were the radius of the axle and CA the radius of the wheel with its handles, it is clear that the power at A would counterpoise the weight at B.Therefore take between B and C any other point such as D, and make BD the radius of the axle and DA the radius of the wheel and handles---

This type of instrument includes the windlass, the capstan, the brace and bit, the wheel with its axle whether the wheel is geared or smooth, and others.

But the brace and bit partakes somewhat of the screw, because when the weight moves (that is, when it makes a hole) by its very nature it always travels onward; hence the worm of the screw is described around a cone.But since it has a sharp point, it may also be reduced to the wedge.

Comment by Pigafetta

Here the author has given us five figures, representing five instruments for moving weights which may be reduced under a single property, in order that one may see each to be the same as the wheel and axle already explained.

[Figure 96]

He has put the letters A, B, and C with their lines, in order that one may understand that the weight has the same proportion to the power that sustains it as AC has to CB, and, as the weight shall be moved by a power, the space of the power will likewise be to the space of the weight as AC is to CB.In each case the power is to be understood as placed at the end of the handles at the distance CA from the center.The weight is to be understood as tied to a rope wound around the axle, at the distance CB from the center.And thus, for the reasons given above, the power that sustains will have the same ratio to the weight as CB has to CA.Similarly the figure with the drum is to be considered as if the force were at the outside of the drum and the weight were attached to the axle.As to the bit and brace, or auger, as it is called, since this is an instrument not designed to sustain things but to move them, the power must have a greater proportion to the weight than that of CB to CA, in accordance with the eleventh proposition in the section on the lever.

End of the Wheel and Axle

On the Wedge

Aristotle in the seventeenth of his Questions of Mechanics declares that the wedge performs the function of two opposed levers in the manner indicated below.

--- Let the wedge be struck as usual on AC; AB is a lever whose fulcrum is at H and weight at B, and in the same way CB is a lever whose fulcrum is at K and weight similarly at B.But when the wedge is struck, it enters into DEFG in a ratio greater than that which was before; let this be the portion MBL.And since MB and BL are greater than HB and BK, ML will also be greater than HK---

[Figure 97]

But since there are three kinds of levers, as set forth previously, it will be perhaps more convenient to consider the wedge in the following manner .

We may regard AB as a lever with its fulcrum at B and the weight at H --- and similarly the lever CB with its support at B and the weight at K---

Thus let the wedge be ABC, and let there be two separate weights DEFG and HIKL with the part DBH of the wedge between them---Now while DG is moved by the wedge toward M --- it is by the lever AB with fulcrum B--- Similarly HL is moved from H by the lever CB---

[Figure 98]

If one must split the rectangle ABCD and there are two equal levers EF and GF--- and it is necessary with these levers to split ABCO without striking, then the moving powers at E and G are equal.

[Figure 99]

But since the whole wedge is moved in the splitting, we may consider it also in another manner; that is, when it enters into the thing to be split and is the same as moving a weight upon an inclined plane.

--- In this example, if we consider the wedge as moving like a lever, it is evident that the wedge BCD moves the weight AEFC by means of a lever CD, so that D is the fulcrum and the weight is placed at E, rather than with the lever BD with its support at H and the weight placed at D.But in order that this may be more clear we shall use another example.

[Figure 100]

--- This movement is easily reduced to the balance and to the lever, since that which is moved on an inclined plane is reduced to the balance by the-ninth proposition of the eighth book of the Mathematical Collections of Pappus.For it is the same thing whether the wedge stands still-and the weight moves on the side of the wedge, or the wedge is moved and the weight moves along its side, as upon an inclined plane.

Comment by Pigafetta

[Figure 101]

The proposition from Pappus, cited here by our author, I have withheld for a more convenient place in the section on the screw, for it is my opinion that perhaps it is more apropos there and serves more clearly than with regard to the wedge.This proposition was sent to me by the author, and, though there was nothing wrong with it, I have compared it carefully with the Greek edition of Pappus owned by Signor Pinelli, that it might be most useful and pleasant to those who have never seen anything of Pappus and have never read that marvelous writer on mechanics.

[Figure 102]

Now we shall see how things that are split move as upon inclined planes.

Next we shall consider what two things are necessary in order that the thing may be more easily moved, or split.

First, as to things being more easily split, the chief essential of the wedge is the angle at the point, for, the more acute the angle, the more easily the wedge moves and splits.

We can also show this by another theory, considering the wedge as moving by two opposed levers, as mentioned previously.

[Figure 103]

Let there be the lever AB that has its fixed support at B and must move the rectangle CDEF, so arranged that it cannot move downward on the side FE.Let the point E be motionless, and consider it as a center, so that the point D will move along the circumference of the circle DH, whose center is E, while C moves along the circumference CL, and the line CE is its radius---Now let there be another lever MCN, which also moves CDEF, and this has its fixed support at N---I say that CDEF is more easily moved by the same power with the lever AB than with the lever MN---

[Figure 104]

Corollary.ÑFrom this it is clear that the smaller the angle BCF or BCE or BCD, the more easily the weight is moved.But this may be demonstrated in the same manner.

The second thing is the reason why a thing is split more easily by striking in order to move the wedge.

Let there be the wedge A which has to split B, and let this be struck by C, which moves and strikes either by itself or by some power that governs and moves it.If by itself, first one must notice that the heavier it is, the greater the stroke will be .In addition to this, the greater the distance AC, the greater the stroke; for any heavy object when moved takes on more heaviness than when standing still, and the more so, the farther it moves .

[Figure 105]

Now if C is moved by some power, as, for example, the handle DE, then first by the greater weight of C, and second by the greater length of DE, the stroke will be made greater.For if the moving power is placed at E, C will be more distant from the center and therefore will move more, as Aristotle demonstrates in his Questions of Mechanics.And it may also be clear from what was said in the section on the balance that the farther C is from the center, the more it will weigh, and it will strike with greater impetus, the force at E being more potent.

[Figure 106]

Now here is the second thing, which is the reason that great weights can be moved and split with this instrument. Percussion is a very strong force, as is evident from the nineteenth of Aristotle's Questions of Mechanics; for if a very heavy weight shall be placed upon a wedge, the wedge will accomplish nothing compared with its [work by] being struck. And even if one were to add a lever or a screw to the wedge, or some other instrument to drive the wedge or screw into the weight, nothing would happen of any importance compared with [that caused by] a stroke.Thus if the body A were a stone from which someone wanted to remove a certain part, say the corner B, then he might break it easily with an iron hammer and without any other instrument; but this he could do only with great difficulty by means of an instrument such as the lever or screw which does not use percussion.Thus percussion is the cause that great weights are split, and if to the great force of percussion we add some instrument suitable for moving and splitting, we shall achieve marvelous things. This instrument is the wedge, of which two properties as to its form must be considered: one, that the wedge is suited to receive and stand the blow; the other, that by the thinness of one of its edges it may easily enter into bodies.Thus the wedge is operated by a blow, and we see almost miracles in the splitting of bodies.To the same property as that of this instrument one may also easily reduce all those things that cut, divide, make holes, or do other things by means of percussion, such as spades, swords, knives, and the like.The saw may also be reduced to this, because its teeth strike and resemble wedges.

End of the Wedge

[Figure 107]

Pappus in his eighth book deals with many matters of the screw, showing how it should be made and how great weights may be moved with such an instrument; moreover, he gives many useful theories for its understanding.Now, since among other things he promises to show that the screw is nothing but a wedge used without percussion, which makes its movements by means of a lever, and this is lacking in his book, we shall attempt to show this and, moreover, to reduce the screw to the lever and the balance in order that ultimately we shall understand it completely.

PROPOSITION I

[Figure 108]

If the wedge is adapted in the following manner to the cylinder, it is precisely a screw which has two worms joined together at one point.

Corollary.ÑFrom this it is evident how one may describe the worms on the screw.

[Figure 109]

We shall now show how weights are moved on the worms of the screw .

[Figure 110]

[Page 63]

Now if to the screw in the next diagram below is applied the gear C with twisted teeth, as Pappus shows in the same eighth book, or even with straight teeth made in such a manner as to fit the screw, it is evident that, with the movement of the screw, the gear C will also turn, and the teeth of the gear will move on the worms on the screw; and this is called the perpetual screw, because both the screw and the wheel will go on turning in the same way.

PROPOSITION II

[Figure 111]

Let there be the screw AB with the worm CDEFC; I say that this is nothing but an inclined plane wound around a cylinder .

But as to the manner in which this is reducible to the balance, that is evident from the ninth proposition of the eighth book of Pappus.

Comment by Pigafetta

The author, in all his books on mechanics, has wished not to insert anything said by others, or which is not entirely his own.Therefore he has omitted the proposition of Pappus which he cites here.But this is so admirably relevant to the explanation of what he says here that I have judged it advisable to add this.

The Problem of Pappus of Alexandria in the Eighth Book of His Mechanical Collections

[Figure 112]

A given force is needed to draw a given weight along a horizontal plane.It is required to find the force needed to draw the weight up another plane inclined at a given angle to the horizontal plane .

Let the horizontal plane pass through MN, and let the plane inclined to the horizontal at the given angle KMN pass through MK.Let A be the weight and C the force required to move it over the horizontal plane.Consider a sphere on the inclined plane passing through M and K.The sphere will be tangent to the plane at L, [as shown in the third theorem of the Spherics of Theodosius].EL will therefore be perpendicular to the plane (for this is [also] shown in the Spherics, Theorem IV), and also to KM.Draw EH parallel to MN, and draw LF from L perpendicular to EH.Now since the angles EHL and KMN are given equal, angle ELF is also given, for the angles ELF and EHL are equal (since triangles EHL and ELF are similar).Therefore the triangle ELF is given in form.Hence the ratio EL : EF, that is, EH : HF, is known, as is also (EH -EF) : EF, that is, HF : EF.

Let weight A be to weight B and force C to force D, as HF is to FE.Now C is the force required to move A.Therefore the force required to move B on the same plane will be D.Since weight A: weight B equals HF : FE, it follows that if E and H are the centers of gravity of weights A and B respectively, the weights will be in equilibrium if balanced at the point F. But weight A has its center of gravity at E (for the sphere represents A).Therefore if weight B is placed so that its center is at H, it will so balance the sphere that the latter will not move down because of the slope of the plane, but will remain unmoved, as if it were on the horizontal plane.But weight A required force C to move it in the horizontal plane.Therefore , to be moved up the inclined plane it will require a force which is the sum of the forces C and D, where D is the force required to move the weight B in the horizontal plane---

We must now consider the two reasons for which weights are easily moved by this instrument.

First, what makes the weight easy to move and what especially belongs to the nature of the screw is the worm; for, if around a given screw AB there should be two unequal worms CDA and EFG, I say that the same weight is moved more easily on CDA than on EFG .

[Figure 113]

The other reason for which weights are easily moved consists in the [length of] the radius or handles by which the screw is turned.

[Figure 114]

Corollary.ÑFrom these things it is evident that the more turns there are to the worm, and the longer the rods or handles, the more easily, but more slowly, the weight is moved.And finally we shall explain the strength of the power that moves it as applied to the rods.

Corollary.ÑFrom this it is clear that a given weight can be moved by a given power by means of the screw.

Thus it has been demonstrated that a weight is moved by the screw as by a wedge without percussion; for, in place of a blow, it moves by means of a lever, that is, the rod or handle.

These things demonstrated, it is evident one can move a given weight by a given power.For if we wish to achieve the effect by a lever, we can with a given lever lift a given weight with a given power.This cannot be done entirely by any of the other devices, whether by the screw, or the wheel and axle, or the pulley; for with a given pulley [system], wheel and axle, or screw, in order to move a given weight, the [required] power is always determined.Therefore, if the power that must move the weight is less than this, it will never move the weight.Yet given the wheel and axle (without handles), we can move a given weight with a given power, since we can arrange handles in such a way that the radius of the wheel together with the length of the handle has the given ratio to the radius of the axle.Now this can also be done with the screw; that is, a given weight can be moved with a given screw by means of the given power.For, if the power is known which must move the weight on the worm, we may arrange the handle in such a way that the given power on the handle has the same force as the power moving the weight on the worm.Although this cannot be done with a given pulley system , yet we can move a given weight with given pulleys by a given power in an infinite number of ways.With the wedge, it seems clear that one can never move a given weight by means-of a given power, because a given power cannot move a given weight along an inclined plane.Nor by a given power can a given weight be moved by opposed levers, such as those of the wedge, inasmuch as the levers in the wedge cannot maintain the true and natural ratio of the lever because the fulcrums of the levers are not motionless when the entire wedge moves.

Anyone will be able then to construct machines and compound several together, such as pulleys and windlasses, or many gears, or in various other ways, and from what we have said one may easily find the relation between the weight and the power.

Comment by Pigafetta

Here one may note that the author has not gone into these last instrumentsÑthat is, the wedge and the screwÑas he did the lever, the pulley, and the wheel and axle, for which he has very exactly shown the ratio of the force and weight.This is because these two instruments in themselves are not suitable to be considered as sustaining a weight, but rather as moving it.Now, since the powers that move may be infinite, one cannot give a firm rule for them as may be done for the power that sustains, which is unique and determined.That the wedge- is not suitably considered as sustaining is clear in itself, and that the same is true of the screw is evident in the ordinary uses of the screw to move weights--- Thus the author has dealt with the two last instruments as suits their nature, as he says in comparing together all five of the instruments for moving weights as a conclusion to his work.

The End