Archimedes, Natation of bodies, 1662

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## HIS TRACT

De Incidentibus Humido, OR OF THE
NATATION OF BODIES VPON, OR SVBMERSION IN,

## THE

## WATER

OR OTHER LIQUIDS.
IN TWO BOOKS.
Tranflated from the Original Greek,
Firft into Latine, and afterwards into Italian, by NICOLO
TARTAGLIA, and by him familiarly demon ftrated by way of Dialogue, with Richard Wentworth, a Noble Englifh Gentleman, and his Friend.
Together with the Learned Commentaries of Federico
Commandino, who hath Reftored fuch of the Demonftrations as, thorow the Injury of Time, were obliterated.
Now compared with the ORIGINAL, and Englifhed By THOMAS SALVSBVRY, Efque
LONDON, Printed by W. Leybourn, 1662.

## HIS TRACT

De
INCIDENTIBUS HUMIDO,
OR OF
The Natation of Bodies upon, or Submerfion in, the Water, or other Liquids.

Dear Companion, I have perufed your Induftrious Invention, in which I find not any thing that will not certainly hold true; but, truth is, there are many of your Conclufions of which I underftand uot the Caufe, and therefore, if it be not a trouble to you, I would defire you to declare them to me, for, indeed, nothing pleafeth me, if the Caufe thereof be hid from me.

NICOLO. My obligations unto you are fo many and great, Honoured Campanion, that no requeft of yours ought to be troublefome to me, and therefore tell me what thofe Perticulars are of which you know not the Caufe, for I fhall endeavour with the utmoft of my power and underftanding to fatisfie you in all your demands.

RIC. In the firft Direction of the firft Book of that your Induffrious Invention you conclude, That it is impoffible that the Water fhould wholly receive into it any material Solid Body that is lighter than it feif (as to fpecix) nay, you fay, That there will alwaies a part of the Body ftay or remain above the Waters Surface (that is uncovered by it;) and, That as the whole Solid Body put into the Water is in proportion to that part of it that fhall be immerged, or received, into the Water, fo fhall the Gravity of the Water be to the Gravity (in fpecix) of that fame material Body: And that thofe Solid Bodies, that are by nature more Grave than the Water, being put into the Water, fhall prefently make the faid Water give place; and, That they do not only wholly enter or fubmerge in the fame, but go continually defcending untill they arrive at the Bottom; and, That they fink to the Bottom fo much fafter, by how much they are more Grave than the Water. And, again, That thofe which are precifely of the fame Gravity with the Water, being put into the fame, are of neceffity wholly received into, or immerged by it, but yet retained in the Surface of the faid Water, and much lefs will the Water confent that it do defcend to the Bottom: and, now, albeit that all thefe things are manifeft to Senfe and Experience, yet neverthelefs would I be very glad, if it be poffible, that you would demonftrate to me the moft apt and proper Caufe of thefe Effects.

NIC. The Caufe of all thefe Effects is affigned by Archimedes, the Siracufan, in
that Book De Incidentibus (^\{*\}) Aqux, by me publifhed in Latine, and dedicated to your felf, as I alfo faid in the beginning of that my Induftrions Invention.

* Aqux, tanflated by me Humido, as the more Comprehenfive word, for his Doctrine holds true in all Liquids as well as in Water, foil. in Wine, Oyl, Milk, \&c.

RIC. I have feen that fame Archimedes, and have very well underftood thofe two Books in which he treateth De Centro Gravitatis æquerepentibus, or of the Center of Gravity in Figures plain, or parallel to the Horizon; and likewife thofe De Quadratura Parabolx, or, of Squaring the Parabola; but $\wedge\{*\}$ that in which he treateth of Solids that Swim upon, or fink in Liquids, is fo obfcure, that, to fpeak the truth, there are many things in it which I do not underftand, and therefore before
we proceed any farther, I fhould take it for a favour if you would declare it to me in your Vulgar Tongue, beginning with his firft Suppofition, which fpeaketh in this manner.

* He fpeaks of but
one Book, Tartag-
lia having tranfla-
ted no more.


## SVPPOSITION I.

It is fuppofed that the Liquid is of fuch a nature, that
its parts being equi-jacent and contiguous, the lefs preffed are repulfed by the more preffed. And that each of its parts is preffed or repulfed by the Liquor that lyeth over it, perpendicularly, if the Liquid be defcending into any place, or preffed any whither by another.

NIC. Every Science, Art, or Doctrine (as you know, Honoured Companion,) hath its firft undemonftrable Principles, by which (they being granted or fuppofed) the faid Science is proved, maintained, or demonftrated. And of thefe Principles, fome are called Petitions, and others Demands, or Suppofitions. I fay, therefore, that the Science or Doctrine of thofe Material Solids that Swim or Sink in Liquids, hath only two undemonftrable Suppofitions, one of which is that above alledged, the which in compliance with your defire I have fet down in our Vulgar Tongue.

RIC. Before you proceed any farther tell me, how we are to underftand the parts of a Liquid to be Equijacent.

NIC. When they are equidiftant from the Center of the World, or of the Earth (which is the fame, although $\wedge\{*\}$ fome hold that the Centers of the Earth and Worldare different.)

RIC. I underftand you not unlefs you give me fome Example thereof in
Figure.

* The Coperni-
cans.

NIC. To exemplifie this particular, Let us fuppofe a quantity of Liquor (as for inftance of Water) to be upon the Earth; then let us with the Imagination cut the whole Earth together with that Water into two equal parts, in fuch a manner as that the faid Section may pafs $\wedge\left\{{ }^{*}\right\}$ by the Center of the Earth: And let us fuppofe that one part of the Superficies of that Section, as well of the Water as of the Earth, be the Superficies A B, and that the Center of the Earth be the point K. This being done, let us in our Imagination defcribe a Circle upon the
faid Center K, of fuch a bignefs as that the Circumference may pafs by the Superficies of the Section of the Water: Now let this Circumference be E F G: and let many Lines be drawn from the point K to the faid Circumference, cutting the fame, as KE, KHO, KFQ KLP, KM. Now I fay, that all thefe parts of the faid Water, terminated in that Circumference, are Equijacent, as being all
equidiftant from the point K , the Center of the World, which parts are G M, M L, L F, F H, H E.

* Or through.

RIC. I underftand you very well, as to this particular: But tell me a little; he faith that each of the parts of the Liquid is preffed or repulfed by the Liquid that is above it, according to the Perpendicular: I know not what that Liquid is that lieth upon a part of another Perpendicularly.

NIC. Imagining a Line that cometh from the Center of the Earth penetrating thorow fome Water, each part of the Water that is in that Line he fuppofeth to be preffed or repulfed by the Water that lieth above it in that fame Line, and that that repulfe is made according to the fame Line, (that is, directly towards the Center of the World) which Line is called a Perpendicular; becaufe every
Right-Line that departeth from any point, and goeth directly towards the Worlds Center is called a Perpendicular. And that you may the better underftand me, let
[Figure 1]
us imagine
the Line KHO, and in that
let us imagine
feveral parts, as fuppofe RS,
S T, T V, V H, H O. I fay, that he fuppofeth that the part V H is preffed by
that placed a-
bove it, H O,
according to
the Line OK;
the which
O K, as hath been faid above, is called the Perpendicular paffing thorow thofe two parts. In like manner, I fay that the part TV is expulfed by the part V H , according to the faid Line O K: and fo the part S T to be preffed by T V , according to the faid Perpendicular O K, and R S by S T. And this you are to underftand in all the other Lines that were protracted from the faid Point K , penetrating the faid Water, As for Example, in K G, K M, K L, K F, K E, and infinite others of the like kind.

RIC. Indeed, Dear Companion, this your Explanation hath given megreat fatisfaction; for, in my Judgment, it feemeth that all the difficulty of this Suppofition confifts in thefe two particulars which you have declared to me.

NIC. It doth fo; for having underftood that the parts E H, H F, F L, L M, and MG, determining in the Circumference of the faid Circle are equijacent, it is an eafie matter to underftand the forefaid Suppofition in Order, which faith, That it is fuppofed that the Liquid is of fuch a nature, that the part thereof lefs preffed or thrust is repulfed by the more thruft or preffed. As for example, if the part E H were by chance more thruft, crowded, or preffed from above downwards by the Liquid, or fome other matter that was over it, than the part H F, contiguous to it, it is fuppofed that the faid part H F, lefs preffed, would be repulfed by the faid part E H. And thus we ought to underftand of the other parts equijacent, in cafe that they be contiguous, and not fevered. That each of the parts thereof is preffed and repul. fed by the Liquid that lieth over it Perpendicularly, is manifeft by that which was faid above, to wit, that it fhould be repulfed, in cafe the Liquid be defcending into any place, and thruft, or driven any whither by another.

RIC. I underftand this Suppofition very well, but yet me thinks that before the Suppofition, the Author ought to have defined thofe two particulars, which you firft declared to me, that is, how we are to underftand the parts of the Liquid equijacent, and likewife the Perpendicular.

NIC. You fay truth.
RIC. I have another queftion to aske you, which is this, Why the Author ufeth the word Liquid, or Humid, inftead of Water.

NIC. It may be for two of thefe two Caufes; the one is, that Water being the principal of all Liquids, therefore faying Humidum he is to be underftood to mean the chief Liquid, that is Water: The other, becaufe that all the Propofitions of this Book of his, do not only hold true in Water, but alfo in every other Liquid, as in Wine, Oyl, and the like: and therefore the Author might have ufed the word Humidum, as being a word more general than Aqua.

RIC. This I underftand, therefore let us come to the firft Propofition, which, as you know, in the Original fpeaks in this manner.

## PROP. I. THEOR. I.

If any Superficies fhall be cut by a Plane thorough any Point, and the Section be alwaies the Circumference of a Circle, whofe Center is the faid Point: that Superficies fhall be Spherical.

Let any Superficies be cut at pleafure by a Plane thorow the Point K; and let the Section alwaies defcribe the Circumference of a Circle that hath for its Center the Point K: I fay, that that fame Superficies is Sphærical. For were it poffible that the faid Superficies were not Sphxrical, then all the Lines drawn through the faid Point K unto that Superficies would not be equal, Let therefore A and B be two Points in the faid Superficies, fo that

[Figure 2]
drawing the two Lines $\mathrm{K} A$ and K B, let them, if poffible, be unequal: Then by thefe two Lines let a Plane be drawn cutting the faid Superficies, and let the Section in the Superficies make the Line
D A B G: Now this Line D A B G
is, by our pre-fuppofal, a Circle, and
the Center thereof is the Point K , for fuch the faid Superficies was fuppofed to be. Therefore the two Lines K A and K B are equal:

But they were alfo fuppofed to be unequal; which is impoffible:
It followeth therefore, of neceffity, that the faid Superficies be
Sphærical, that is, the Superficies of a Sphære.
RIC. I underftand you very well; now let us proceed to the fecond Propofition, which, you know, runs thus.

The Superficies of every Liquid that is confiftant and fetled fhall be of a Spharical Figure, which Figure fhall have the fame Center with the Earth.

Let us fuppofe a Liquid that is of fuch a confiftance as that it is not moved, and that its Superficies be cut by a Plane along by the Center of the Earth, and let the Center of the Earth be the Point K: and let the Section of the Superficies be the Line A B G D. I fay that the Line A B G D is the Circumference of a

[Figure 3]

Circle, and that the Center thereof is the Point K And if it be poffible that it may not be the Circumference of a Circle, the Right-

Lines drawn $\wedge\left\{^{*}\right\}$ by the Point
K to the faid Line A B G D
fhall not be equal. There-
fore let a Right-Line be
taken greater than fome of thofe produced from the Point $K$ unto the faid Line A B G D, and leffer than fome other; and upon the Point K let a Circle be defcribed at the length of that Line, Now the Circumference of this Circle fhall fall part without the faid Line A B G D, and part within: it having been prefuppofed that its Semidiameter is greater than fome of thofe Lines that may be drawn from the faid Point K unto the faid Line A B G D, and leffer than fome other. Let the Circumference of the defcribed Circle be R B G H, and from B to K draw the Right-Line B K: and drawn alfo the two Lines K R, and K E L which make a RightAngle in the Point K : and upon the Center K defcribe the Circumference X O P in the Plane and in the Liquid. The parts, therefore, of the Liquid that are $\left.\wedge^{\{ }{ }^{*}\right\}$ according to the Circumference

X O P, for the reafons alledged upon the firft Suppofition, are equijacent, or equipofited, and contiguous to each other; and both thefe parts are preft or thruft, according to the fecond part of the Suppofition, by the Liquor which is above them. And becaufe the two Angles E K B and B K R are fuppofed equal [by the 26. of 3. of Euclid,] the two Circumferences or Arches B E and B R fhall be equal (forafmuch as R B G H was a Circle defcribed for fatis-
faction of the Oponent, and K its Center:) And in like manner the whole Triangle B E K fhall be equal to the whole Triangle B R K. And becaufe alfo the Triangle O P K for the fame reafon
fhall be equal to the Triangle OXK; Therefore (by common Notion) fubftracting thofe two fmall Triangles O P K and O X K from the two others B E K and BRK, the two Remainders fhall be equal: one of which Remainders fhall be the Quadrangle B E O P , and the other B R X O. And becaufe the whole Quadrangle B E O P is full of Liquor, and of the Quadrangle B R X O, the part B A X O only is full, and the refidue B R A is wholly void of Water: It followeth, therefore, that the Quadrangle B E O P is more ponderous than the Quadrangle B R X O. And if the faid Quadrangle B E O P be more Grave than the Quadrangle B R X O, much more fhall the Quadrangle B L O P exceed in Gravity the faid Quadrangle BRXO: whence it followeth, that the part O P is more preffed than the part O X. But, by the firft part of the Suppofition, the part lefs preffed fhould be repulfed by the part more preffed: Therefore the part O X muft be repulfed by the part O P: But it was prefuppofed that the Liquid did not move: Wherefore it would follow that the lefs preffed would not be repulfed by the more preffed: And therefore it followeth of neceffity that the Line A B G D is the Circumference of a Circle, and that the Center of it is the point K. And in like manner fhall it be demonftrated, if the Surface of the Liquid be cut by a Plane thorow the Center of the Earth, that the Section fhall be the Circumference of a Circle, and that the Center of the fame fhall be that very Point which is Center of the Earth. It is therefore manifeft that the Superficies of a Liquid that is confiftant and fetled fhall have the Figure of a Sphære, the Center of which fhall be the fame with that of the Earth, by the firft Propofition; for it is fuch that being ever cut thorow the fame Point, the Section or Divifion defcribes the Circumference of a Circle which hath for Center the felf-fame Point that is Center of the Earth: Which was to be demonftrated.

* O: through.
* i.e. Parallel.

RIC. I do thorowly underftand thefe your Reafons, and fince there is in them no umbrage of Doubting, let us proceed to his third Propofition.

## PROP. III. THEOR. III.

Solid Magnitudes that being of equal Mafs with the Liquid are alfo equal to it in Gravity, being demit-
ted into the [ $\left.\wedge{ }^{*}\right\}$ fetled] Liquid do fo fubmerge in the fame as that they lie or appear not at all above the Surface of the Liquid, nor yet do they fink to the Bottom.
*I add the word
fetled, as neceffary
in making the Experiment.

NIC. In this Propofition it is affirmed that thofe Solid Magnitules that happen to be equal in fpecifical Gravity with the Liquid being lefeat liberty in the faid Liquid do fo fubmerge in the fame, as that they lie or appear not at all above the Surface of the Liquid, nor yet do they go or fink to the Bottom.

For fuppofing, on the contrary, that it were poffible for one of thofe Solids being placed in the Liquid to lie in part without the Liquid, that is above its Surface, (alwaies provided that the faid Liquid be fetled and undifturbed,) let us imagine any Plane produced thorow the Center of the Earth, thorow the Liquid, and thorow that Solid Body: and let us imagine that the Section of the Liquid is the Superficies A B G D, and the Section of the Solid Body that is within it the Superficies E Z H T, and let us fuppofe the Center of the Earth to be the Point K: and let the part of the faid Solid fubmerged in the Liquid be B G H T, and let that above be B E Z G: and let the Solid Body be fuppofed to be comprized in a Pyramid that hath its Parallelogram Bafe in the upper Surface of the Liquid, and its Summity or Vertex in the Center of the Earth: which Pyramid let us alfo fuppofe to be cut or divided by the fame Plane in which is the Circumference A B G D, and let the Sections
of the Planes of the faid Pyramid be K L and
K M : and in the Liquid about the Center K let there be defcribed a Su perficies of another Sphære below E Z H T, which let be X O P; and let this be cut by
the Superficies of the Plane: And let there be another Pyramid taken or fuppofed equal and like to that which comprifeth the faid Solid Body, and contiguous and conjunct with the fame; and let the Sections of its Superficies be K M and K N : and let us fuppofe another Solid to be taken or imagined, of Liquor, contained in that fame Pyramid, which let be R S C Y, equal and like to the partial

Solid B H G T, which is immerged in the faid Liquid: But the part of the Liquid which in the firft Pyramid is under the Superficies X O , and that, which in the other Pyramid is under the Superficies $O P$, are equijacent or equipofited and contiguous, but are not preffed equally; for that which is under the Superficies X O is preffed by the Solid T H E Z, and by the Liquor that is contained between the two Spherical Superficies X O and L M and the Planes of the Pyramid, but that which proceeds according to F O is preffed by the Solid R S C Y, and by the Liquid
contained between the Sphærical Superficies that proceed according to P O and M N and the Planes of the Pyramid; and the Gravity of the Liquid, which is according to M N O P, fhall be leffer than that which is according to L M X O; becaufe that Solid of Liquor which proceeds according to R S C Y is lefs than the Solid E Z H T (having been fuppofed to be equal in quantity to only the part H B G T of that:) And the faid Solid E Z H T hath been fuppofed to be equally grave with the Liquid: Therefore the Gravity of the Liquid comprifed betwixt the two Sphærical Superficies L M and X O , and betwixt the fides L X and M O of the
[Figure 5]

Pyramid, together with
the whole Solid EZHT, fhall exceed the Gravity of the Liquid comprifed betwixt the other two Sphærical Superficies M N and OP , and the Sides MO and NP
of the Pyramid, toge-
ther with the Solid of Liquor R S C Y by the quantity of the Gravity of the part E B Z G, fuppofed to remain above the Surface of the Liquid: And therefore it is manifeft that the part which proceedeth according to the Circumference O P is preffed, driven, and repulfed, according to the Suppofition, by that which proceeds according to the Circumference X O, by which means the Liquid would not be fetled and ftill: But we did prefuppofe that it was fetled, namely fo, as to be without motion: It followeth, therefore, that the faid Solid cannot in any part of it exceed or lie above the Superficies of the Liquid: And alfo that being dimerged in the Liquid it cannot defcend to the Bottom, for that all the parts of the Liquid equijacent, or difpofed equally, are equally preffed, becaufe the Solid is equally grave with the Liquid, by what we prefuppofed.

RIC. I do underftand your Argumentation, but I underftand not that Phrafe Solid Magnitudes.

NIC. I will declare this Term unto you. Magnitude is a general Word that refpecteth all the Species of Continual Quantity; and the Species of Continual Quantity are three, that is, the Line, the Superficies, and the Body; which Body is alfo called a Solid, as having in it felf Length, Breadth, and Thicknefs, or Depth: and therefore that none might equivocate or take that Term Magnitudes to be
meant of Lines, or Superficies, but only of Solid Magnitudes, that is, Bodies, he did fpecifie it by that manner of expreffion, as was faid. The truth is, that he might have expreft that Propofition in this manner: Solids (or Bodies) which being of equal Gravity with an equal Mafs of the Liquid, \&c. And this Propofition would have been more cleer and intelligible, for it is as fignificant to fay, a Solid, or, a Body, as to fay, a Solid Magnitude: therefore wonder not if for the future I ufe thefe three kinds of words indifferently.

RIC. You have fufficiently fatisfied me, wherefore that we may lofe no time let us go forwards to the fourth Propofition.

Solid Magnitudes that are lighter than the Liquid, being demitted into the fetled Liquid, will not totally fubmerge in the fame, but fome part thereof will lie or ftay above the Surface of the Liquid.

NIC. In this fourth Propofition it is concluded, that every Body or Solid that is lighter (as to Specifical Gravity) than the Liquid, being put into the Liquid, will not totally fubmerge in the fame, but that fome part of it will ftay and appear without the Liquid, that is above its Surface.

For fuppofing, on the contrary, that it were poffible for a Solid more light than the Liquid, being demitted in the Liquid to fubmerge totally in the fame, that is, fo as that no part thereof remaineth above, or without the faid Liquid, (evermore fuppofing that the Liquid be fo conftituted as that it be not moved,) let us imagine any Plane produced thorow the Center of the Earth, thorow the Liquid, and thorow that Solid Body: and that the Surface of the Liquid is cut by this Plane according to the Circumference A B G, and the Solid Body according to the Figure R; and let the Center of the Earth be K. And let there be imagined a Pyramid
[Figure 6]
that comprifeth the Figure
$R$, as was done in the pre.
cedent, that hath its Ver-
tex in the Point $K$, and let
the Superficies of that
Pyramid be cut by the
Superficies of the Plane
A B G, according to AK
and K B. And let us ima-
gine another Pyramid equal and like to this, and let its Superficies be cut by the Superficies A B G according to K B and K G; and let the Superficies of another Sphære be defcribed in the Liquid, upon the Center K, and beneath the Solid R; and let that be cut by the fame Plane according to X O P. And, laftly, let us fuppofe another Solid taken $\wedge\{*\}$ from the Liquid, in this fecond Pyramid, which
let be H, equal to the Solid R. Now the parts of the Liquid, namely, that which is under the Spherical Superficies that proceeds according to the Superficies or Circumference X O, in the firft Pyramid, and that which is under the Spherical Superficies that pro-
ceeds according to the Circumference O P, in the fecond Pyramid, are equijacent, and contiguous, but are not preffed equally; for
that of the firft Pyramid is preffed by the Solid R, and by the Liquid which that containeth, that is, that which is in the place of the Pyramid according to A B O X: but that part which, in the other Pyramid, is preffed by the Solid H , fuppofed to be of the fame $\mathrm{Li}^{-}$ quid, and by the Liquid which that containeth, that is, that which is in the place of the faid Pyramid according to P O B G: and the Gravity of the Solid R is lefs than the Gravity of the Liquid H , for that thefe two Magnitudes were fuppofed to be equal in Mafs, and the Solid R was fuppofed to be lighter than the Liquid: and the Maffes of the two Pyramids of Liquor that containeth thefe
two Solids R and H are equal $\wedge\{*\}$ by what was prefuppofed: Therefore the part of the Liquid that is under the Superficies that proceeds according to the Circumference O P is more preffed; and, therefore, by the Suppofition, it fhall repulfe that part which is lefs preffed, whereby the faid Liquid will not be fetled: But it was before fuppofed that it was fetled: Therefore that Solid R fhall not totally fubmerge, but fome part thereof will remain without the Liquid, that is, above its Surface, Which was the Propofition.

* That is a Mafs of the Liquid.
* For that the Py-
ramids were fuppo-
fed equal.
RIC. I have very well underftood you, therefore let us come to the fifth Propofition, which, as you know, doth thus fpeak.


## PROP. V. THEOR. V.

Solid Magnitudes that are lighter than the Liquid, being demitted in the (fetled) Liquid, will fo far fubmerge, till that a Mafs of Liquor, equal to the Part fubmerged, doth in Gravity equalize the whole Magnitude.

NIC. It having, in the precedent, been demonftrared that Solids lighter than the Liquid, being demitted in the Liquid, alwaies a part of them remains without the Liquid, that is above its Surface; In this fifth Propofition it is afferted, that fo much of fuch a Solid fhall fubmerge, as that a Mafs of the Liquid equal to the part fubmerged, fhall have equal Gravity with the whole Solid.

And to demonftrate this, let us affume all the fame Schemes as before, in Propofition 3. and likewife let the Liquid be fet-
led, and let the Solid E Z H T be lighter than the Liquid.
Now if the faid Liquid be fetled, the parts of it that are equija-
cent are equally preffed: Therefore the Liquid that is beneath
the Superficies that proceed according to the Circumferences X O and PO are equally preffed; whereby the Gravity preffed is equal.

[Figure 7]

But the Gravity of the Liquid which is in the
firft Pyramid $\wedge\left\{{ }^{*}\right\}$ without
the Solid B H T G, is equal to the Gravity of the Liquid which is in the other Pyramid without the Liquid R S C Y:
It is manifeft, therefore, that the Gravity of the Solid E Z H T, is equal to the Gravity of the Liquid R S C Y: Therefore it is manifeft that a Mafs of Liquor equal in Mafs to the part of the Solid fubmerged is equal in Gravity to the whole Solid.

* Without, i.e. that
being deducted.
RIC. This was a pretty Demonftration, and becaufe I very well underftand it, let us lofe no time, but proceed to the fixth Propofition, fpeaking thus.

PROP. VI. THEOR. VI.

Solid Magnitudes lighter than the Liquid being thruft into the Liquid, are repulfed upwards with a Force as great as is the excefs of the Gravity of a Mafs of Liquor equal to the Magnitude above the Gravity of the faid Magnitude.

NIC. This fixth Propofition faith, that the Solids lighter than the Liquid demitted, thruft, or trodden by Force underneath the Liquids Surface, are returned or driven upwards with fo much Force, by how much a quantity of the Liquid equal to the. Solid fhall exceed the faid Solid in Gravity.

And to delucidate this Propofition, let the Solid A be lighter than the Liquid, and let us fuppofe that the Gravity of the faid Solid A is B: and let the Gravity of a Liquid, equal in Mafs to A, be B G. I fay, that the Solid A depreffed or demitted with Force into the faid Liquid, fhall be returned and repulfed upwards with
a Force equal to the Gravity G. And to demonftrate this Propofition, take the Solid D, equal in Gravity to the faid G. Now the Solid compounded of the two Solids A and D will be lighter than the Liquid: for the Gravity of the Solid compounded of them both is BG, and the Gravity of as much Liquor as equalleth in greatnefs the Solid A, is greater than the faid Gravity BG,
for that B G is the Gravity of the Liquid equal in Mafs unto it: Therefore the Solid compounded of thofe two Solids A and D being dimerged, it fhall, by the precedent, fo much of it fubmerge, as that a quantity of the Liquid equal to the faid fubmerged part fhall have equal Gravity with the faid compounded Solid. And

[Figure 8]
for an example of that Propofition let the Superficies of any Liquid be that which proceedeth according to the Circumference A B G D: Becaufe now a Mafs or quantity of Liquor as big as the Mafs A hath equal Gravity with the whole compounded Solid A $D$ : It is manifeft that the fubmerged part thereof fhall be the Mafs A: and the remainder, namely, the part D, fhall be wholly atop, that is, above the Surface of the Liquid.
It is therefore evident, that the part A hath fo much virtue or Force to return upwards, that is, to rife from below above the $\mathrm{Li}-$ quid, as that which is upon it, to wit, the part D, hath to prefs it downwards, for that neither part is repulfed by the other: But D preffeth downwards with a Gravity equal to G, it having been fuppofed that the Gravity of that part D was equal to G : Therefore that is manifeft which was to be demonftrated.

RIC. This was a fine Demonftration, and from this I perceive that you collected your Induftrious Invention; and efpecially that part of it which you infert in the firft Book for the recovering of a Ship funk: and, indeed, I have many Queftions to ask you about that, but I will not now interrupt the Difcourfe in hand, but defire that we may go on to the feventh Propofition, the purport whereof is this.

## PROP. VII. THEOR. VII.

Solid Magnitudes beavier than the Liquid, being demitted into the [fetled] Liquid, are boren downwards as far as they can defcend: and fhall be lighter in the Liquid by the Gravity of a Liquid Mafs of the fame bignefs with the Solid Magnitude.

NIC. This feventh Propofition hath two parts to be demonftrated.
The firft is, That all Solids heavier than the Liquid, being demitted into the Liquid, are boren by their Gravities downwards as far as they can defcend, that is untill they arrive at the Bottom. Which
firft part is manifeft, becaufe the Parts of the Liquid, which ftill lie under that Solid, are more preffed than the others equijacent,
becaufe that that Solid is fuppofed more grave than the Liquid.

But now that that Solid is lighter in the Liquid than out of it, as is affirmed in the fecond part, fhall be demonftrated in this manner. Take a Solid, as fuppofe A, that is more grave than the Liquid, and fuppofe the Gravity of that fame Solid A to be BG. And of a Mafs of Liquor of the fame bignefs with the Solid A, fuppofe the Gravity to be B: It is to be demonftrated that the Solid A, immerged in the Liquid, fhall have a Gravity equal to G. And to demonftrate this, let us imagine another Solid, as fuppofe D, more light than the Liquid, but of fuch a quality as that its Gravity is equal to $B$ : and let this D be of fuch a Magnitude, that a Mafs of Liquor equal to it hath its Gravity equal to the Gravity B G. Now thefe two Solids D and A being compounded together, all that Solid compounded of thefe two fhall be equally Grave with the Water: becaufe the Gravity of thefe two Solids together fhall be equal to thefe two Gravities, that is, to B G, and

[Figure 9]
to B; and the Gravity of a Liquid that hath its Mafs equal to thefe two Solids A and D, fhall be equal to thefe two Gravities B G and B. Let thefe two Solids, therefore, be put in the Liquid,
and they fhall $\wedge\{*\}$ remain in the Surface of that Li-
quid, (that is, they fhall not be drawn or driven
upwards, nor yet downwards:) For if the Solid
A be more grave than the Liquid, it fhall be
drawn or born by its Gravity downwards to-
wards the Bottom, with as much Force as by the Solid D it is thruft upwards: And becaufe the Solid D is lighter than the Liquid, it fhall raife it upward with a Force as great as the Gravity G: Becaufe it hath been demonftrated, in the fixth Propofition, That Solid Magnitudes that are lighter than the Water, being demitted in the fame, are repulfed or driven upwards with a Force fo much the greater by how much a Liquid of equal Mafs with the Solid is more Grave than the faid Solid: But the Liquid which is equal in Mafs with the Solid D, is more grave than the faid Solid D, by the Gravity G: Therefore it is manifeft, that the Solid A is preffed or born downwards towards the Centre of the World, with a Force as great as the Gravity G : Which was to be demonftrated.

* Or, according to

Commandine, fhall
be equall in Gravi-
ty to the Liquid,
neither moving up-
wards or down-
wards.
RIC. This hath been an ingenuous Demonftration; and in regard I do fufficiently underftand it, that we may lofe no time, we will proceed to the fecond Suppofition, which, as I need not tell you, fpeaks thus.

It is fuppofed that thofe Solids which are moved upwards, do all afcend according to the Perpendicular which is produced thorow their Centre of Gravity.

## COMMANDINE.

And thofe which are moved downwards, defcend, likewife, according to the Perpendicular that is produced thorow their Centre of Gravity, which he pretermitted either as known, or as to be collected from what went before.

NIC. For underftanding of this fecond Suppofition, it is requifite to take notice that every Solid that is lighter than the Liquid being by violence, or by fome other occafion, fubmerged in the Liquid, and then left at liberty, it fhall, by that which hath been proved in the fixth Propofition, be thruft or born up wards by the Liquid, and that impulfe or thrufting is fuppofed to be directly according to the Perpendicular that is produced thorow the Centre of Gravity of that Solid; which Perpendicular, if you well remember, is that which is drawn in the Imagination from the Centre of the World, or of the Earth, unto the Centre of Gravity of that Body, or Solid.

RIC. How may one find the Centre of Gravity of a Solid?
NIC. This he fheweth in that Book, intituled De Centris Gravium, vel de Æquiponderantibus; and therefore repair thither and you fhall be fatisfied, for to declare it to you in this place would caufe very great confufion.

RIC. I underftand you: fome other time we will talk of this, becaufe I have a mind at prefent to proceed to the laft Propofition, the Expofition of which feemeth to me very confufed, and, as I conceive, the Author hath not therein fhewn all the Subject of that Propofition in general, but only a part: which Propofition fpeaketh, as you know, in this form.

## PROP. VIII. THEOR. VIII.

## A

If any Solid Magnitude, lighter than the Liquid, that hath the Figure of a Portion of a Sphære, fhall be
demitted into the Liquid in fuch a manner as that the Bafe of the Portion touch not the Liquid, the Figure fhall ftand erectly, fo, as that the Axis of the faid Portion fhall be according to the Perpendicular. And if the Figure fhall be inclined to any fide, fo, as that the Bafe of the Portion touch the Liquid, it fhall not continue fo inclined as it was demitted, but fhall return to its uprightnefs.

For the declaration of this Propofition, let a Solid Magnitude that hath the Figure of a portion of a Sphære, as hath been faid, be imagined to be de-

[Figure 10]
mitted into the Liquid; and alfo, let a Plain be fuppofed to be produced thorow the Axis of that portion, and thorow the Center of the
Earth: and let the Section of the Surface of the Liquid be the Circumference A B
C D, and of the Figure, the
Circumference E F H, \& let
E H be a right line, and F T
the Axis of the Portion. If now
it were poffible, for fatisfact-
ion of the Adverfary, Let
it be fuppofed that the faid Axis were not according to the (a) Per-
pendicular; we are then to demonftrate, that the Figure will not continue as it was conftituted by the Adverfary, but that it will return, as hath been faid, unto its former pofition, that is, that the Axis F T fhall be according to the Perpendicular. It is manifeft, by the Corollary of the 1 . of 3 . Euclide, that the Center of the Sphære is in the Line F T, forafmuch as that is the Axis of that Figure.
And in regard that the Por-

[Figure 11]
tion of a Sphrre, may be
greater or leffer than an He-
mifphære, and may alfo be
an Hemifphære, let the Cen-
tre of the Sphære, in the He-
mifphære, be the Point T,
and in the leffer Portion the Point P , and in the greater, the Point $K$, and let the Centre of the Earth be the Point
L. And fpeaking, firft, of that greater Portion which
hath its Bafe out of, or a-
bove, the Liquid, thorew the Points K and L , draw the Line KL cutting the Circumference E F H in the Point N, Now, becaufe
every Portion of a Sphære, hath its Axis in the Line, that from the Centre of the Sphære is drawn perpendicular unto its Bafe, and hath its Centre of Gravity in the Axis; therefore that Portion of the Figure which is within the Liquid, which is compounded of two
tions of a Sphxre, fhall have its Axis in the Perpendicular, that is drawn through the point K ; and its Centre of Gravity, for the fame reafon, fhall be in the Line N K: let us fuppofe it to be the Point R:

But the Centre of Gravity of the whole Portion is in the Line F T, betwixt the Point R and

[Figure 12]
the Point F; let us fuppofe it to be the Point X : The remainder, therefore, of that

Figure elivated above the Surface of the Liquid, hath its Centre of Gravity in the Line R X produced or continued right out in the Part towards X, taken fo, that the part prolonged may have the fame proportion to X R, that the Gravity of that Portion that is demer-
ged in the Liquid hath to
the Gravity of that Figure which is above the Liquid; let us fuppofe
that $\left.\wedge^{\wedge}{ }^{*}\right\}$ that Centre of the faid Figure be the Point S : and thorow that
fame Centre S draw the Perpendicular L S. Now the Gravity of the Figure that is above the Liquid fhall preffe from above downwards according to the Perpendicular S L; \& the Gravity of the Portion that is fubmerged in the Liquid, fhall preffe from below upwards, according to the Perpendicular R L. Therefore that Figure will not continue according to our Adverfaries Propofall, but thofe parts of the faid Figure which are towards E, fhall be born or drawn downwards, \& thofe which are towards H fhall be born or driven upwards, and this fhall be fo long untill that the Axis F T comes to be according to the Perpendicular.
(a) Perpendicular is taken kere, as in all other places, by this Author for the Line K L
drawn thorow the
Centre and Cir-
cumference of the Earth.

C

D

E

* i. e, The Center
of Gravity.


## F

And this fame Demonftration is in the fame manner verified in the other Portions. As, firft, in the Hæmifphere that lieth with its whole Bafe above or without the Liquid, the Centre of the Sphære hath been fuppofed to be the Point T; and therefore, imagining T to be in the place, in which, in the other above mentioned, the Point R was, arguing in all things elfe as you did in that, you fhall find that the Figure which is above the Liquid fhall prefs from above downwards according to the Perpendicular S L; and the Portion that is fubmerged in the Liquid fhall prefs from below upwards according to the Perpendicular R L. And therefore it fhall follow, as in the other, namely, that the parts of the whole Figure which are towards E, fhall be born or preffed downwards, and thofe
that are towards H , fhall be born or driven upwards: and this fhall be fo long untill that the Axis F T come to ftand $\left.\wedge^{\wedge}{ }^{*}\right\}$ P
ly. The like fhall alfo hold true in the Portion of the Sphære
lefs than an Hemifphere that lieth with its whole Bafe above the Liquid.

* Or according to the Perpendi-
cular.


## COMMANDINE.

The Demonftration of this Propofition is defaced by the Injury of Time, which we have reftored, fo far as by the Figures that remain, one may collect the Meaning of Archimedes, for we thought it not good to alter them: and what was wanting to their declaration and explanation we have fupplyed in our Commentaries, as we have alfo determined to do in the fecond Propofition of the fecond Book.

If any Solid Magnitude lighter than the Liquid.] Thefe words, light-
er than the Liquid, are added by us, and are not to be found in the Tranfiation; for of thefe kind of Magnitudes doth Archimedes fpeak in this Propofition.

## A

Shall be demitted into the Liquid in fuch a manner as that the

Bafe of the Portion touch not the Liquid.] That is, fhall be fo demitted into the Liquid as that the Bafe fhall be upwards, and the Vertex downwards, which he oppofeth to that which he faith in the Propofition following; Be demitted into the Liquid, fo, as that its Bafe be wholly within the Liquid; For thefe words fignifie the Portion demitted the contrary way, as namely, with the Vertex upwards and the Bafe downwards. The fame manner of fpeech is frequently ufed in the fecond Book; which treateth of the Portions of Rectangle Conoids.

B
Now becaufe every Portion of a Sphære hath its Axis in the Line that from the Center of the Sphære is drawn perpendicular to its Bafe.] For draw a Line from B to C, and let K L cut the Circumference A B C D in the Point G, and the Right Line B C in M:

[Figure 13]
and becaufe the two Circles A B C D, and E F H do cut one another in the Points
B and C, the Right Line that conjoyneth
their Centers, namely, K L, doth cut the Line B C in two equall parts, and at Right Angles; as in our Commentaries upon Prolomeys Planifphære we do prove: But of the Portion of the Circle B N C the Diameter is M N; and of the Portion B G C the Diameter is M G;
for the (a) Right Lines which are drawn on both fides parallel to B C do make

Right Angles with N G; and (b) for that caufe are thereby cut in two equall parts: Therefore the Axis of the Portion of the Sphære B N C is N M; and the Axis of the Portion B G C is M G: from whence it followeth that the Axis of the Portion demerged in the Liquid is in the Line K L, namely N G. And fince the Center of Gravity of any Portion of a Sphrre is in the Axis, as we have demonstrated in our Book De Centro Gravitatis Solidorum, the Centre of Gravity of the Magnitude compounded of both the Portions B N C \& B G C, that is, of the Portion demerged in the Water, is in the Line N G that doth conjoyn the Centers of Gravity of thofe Portions of Sphxres. For fuppofe, if poffible, that it be out of the Line N G, as in Q , and let the Center of the Gravity of the Portion B N C, be V, and draw V que Becaufe therefore from the Portion demerged in the Liquid the Portion of the Sphxre B N C, not having the fame Center of Gravity, is cut off, the Center of Gravity of the Remainder of the Portion B G C fhall, by the 8 of the firft Book of Archimedes, De Centro Gravitatis

Planotum, be in the Line $V \mathrm{Q}$ prolonged: But that is impoffible; for it is in the Axis G: It followeth, therefore, that the Center of Gravity of the Portion demerged in Liquid be in the Line N K : which we propounded to be proved.

## C

(a) By 29. of the
firft of Encl.
(b) By 3. of the third.

But the Centre of Gravity of the whole Portion is in the Line
T, betwixt the Point R and the Point F; let us fuppofe it to be the Point X.] Let the Sphære becompleated, fo as that there be added of that Portion the Axis T Y, and the Center of Gravity Z. And becaufe that from the whole Sphære, whofe Centre of Gravity is $K$, as we have alfo demonftrated in the (c) Book before named, the is cut off the Portion E Y H, having the Centre of Gravity Z; the Centre of the remaind
of the Portion E F H fhall be in the Line Z K prolonged: And therefore it muft of neceffity fall betwixt K and F .

D
(c) By 8 of the
firft of Archimedes.

## E

The remainder, therefore, of the Figure, elevated above the Surface of the Liquid, hath its Center of Gravity in the Line R X prolonged.] By the fame 8 of the firft Book of Archimedes, de Centro Gravitatis Planorum.

Now the Gravity of the Figure that is above the Liquid fhall prefs from above downwards according to S L; and the Gravit of the Portion that is fubmerged in the Liquid fhall prefs from be low upwards, according to the Perpendicular R L.] By the fecond Suppofition of this. For the Magnitude that is demerged in the Liquid is moved upwards with as much Force along R L, as that which is above the Liquid is moved downwards along S L; as may be fhewn by Propofition 6. of this. And becaufe they are moved along feverall other Lines, neither caufeth the others being lefs moved; the which it continually doth when the Portion is fet according to the Perpendicular: For then the Centers of Gravity of both the Magnitudes do concur in one and the fame Perpendicular, namely, in the Axis of the Portion: and look with what force or Impetus that which is in the Lipuid tendeth upwards, and with the like doth that which is above or without the Liquid tend downwards along the fame Line: And therefore, in regard that the one doth not $\left.\wedge^{\wedge}{ }^{*}\right\}$ exceed the other, the Portion fhall no longer move
but fhall ftay and reft allwayes in one and the fame Pofition, unlefs fome extrinfick Caufe chance to intervene.

F

* Or overcome.


## PROP. IX. THEOR. IX.

* In fome Greek

Coppies this is no
diftinct Propofi-
tion, but all
Commentators, do divide it
from the Prece-
dent, as having a
diftinct demon-
fration in the
Originall.
$\wedge\{*\}$ But if the Figure, lighter than the Liquid, be demitted into the Liquid, fo, as that its Bafe be wholly within the faid Liquid, it fhall continue in fuch manner erect, as that its Axis fhall ftand according to the Perpendicular.

For fuppofe, fuch a Magnitude as that aforenamed to be de mitted into the Liquid; and imagine a Plane to be produced thorow the Axis of the Portion, and thorow the Center of the Earth: And let the Section of the Surface of the Liquid, be the Circumference A B C D, and of the Figure the Circumference E F H And let E H be a Right Line, and F T the Axis of the Portion. If now it were poffible, for fatisfaction of the Adverfary, let it be fuppofed that the faid Axis were not according to the Perpendicular: we are now to demonftrate that the Figure will not fo
nue, but will return to be according to the

[Figure 14]

Perpendieular. It is manifeft that the Gentre of the Sphære is in the Line F T. And again, forafmuch as the Portion of a Sphære may be greater or leffer than an Hemifphære, and may alfo be an Hemifphære, let the Centre of the Sphære in the Hemifphære be the Point T, \& in the leffer Portion the Point P , and in the Greater the

Point R. And fpeaking firft of that greater Portion which hath its Bafe within the

Liquid, thorow R and L, the Earths Cen-

[Figure 15]
tre, draw the line RL. The Portion that is above the Liquid, hath its Axis in the Perpendicular paffing thorow R ; and by what hath been faid before, its Centre of Gravity fhall be in the Line N R; let it be the Point R: But the Centre of Gravity of the whole Portion is in the line F T , betwixt R and F ; let it be X : The remainder therefore of that Figure, which is within the Liquid fhall have its Centre in the Right Line R X prolonged in the part

[Figure 16]
towards X , taken fo, that the part prolonged may have the fame Proportion to X R, that the Gravity of the Portion that
is above the Liquid hath to the Gravity of the Figure that is within the Liquid. Let O be the Centre of that fame Figure: and thorow O draw the Perpendicular L O. Now the Gravity of the Portion that is above the Liquid fhall prefs according to the Right Line R L downwards; and the Gravity of the Figure that is in the Liquid according to the Right Line O L upwards: There the Figure fhall not continue; but the parts of it towards H fhall move downwards, and thofe towards E upwards: \&

[Figure 17]
this fhall ever be, fo long as F T is according to the Perpendicular.

## A

## COMMANDINE.

The Portion that is above the Liquid
hath its Axis in the Perpendicular paffing
thorow K.] For draw B C cutting the Line N K in
M ; and let N K out the Circumference A B C D in G. In
the fame manner as before me will demonftrate, that the Axis
of the Portion of the Sphære is N M; and of the Portion B G C the Axis is G M: Wherefore the Centre of Gravity of them both fhall be in the Line N M: And becaufe that from the Portion B N C the Portion B G C, not having the fame Centre of Gravity, is cut off, the Centre of Gravity of the remainder of the Magnitude that is above the Surface of the Liquid fhall be in the Line N K; namely, in the Line which conjoyneth the Centres of Gravity of the faid Portions by the forefaid 8 of Archimedis de Centro Gravitatis Planorum.

## A

NIC. Truth is, that in fome of thefe Figures $C$ is put for X , and fo it was in the Greek Copy that I followed.

RIC. This Demoftration is very difficult, to my thinking; but I believe that it is becaufe I have not in memory the Propofitions of that Book entituled De Centris Gravium.

NIC. It is fo.

RIC. We will take a more convenient time to difcourfe of that, and now return
to fpeak of the two laft Propofitions. And I fay that the Figures incerted in the demonftration would in my opinion, have been better and more intelligble unto me, drawing the Axis according to its proper Pofition; that is in the half Arch of thefe Figures, and then, to fecond the Objection of the Adverfary, to fuppofe that the faid Figures ftood fomewhat Obliquely, to the end that the faid Axis, if it were poffible, did not ftand according to the Perpendicular fo often mentioned, which doing, the Propofition would be proved in the fame manner as before: and this way would be more naturall and clear.

A

B

NIC. You are in the right, but becaufe thus they were in the Greek Copy, I thought not fit to alter them, although unto the better.

RIC. Companion, you have thorowly fatisfied me in all that in the beginning of our Difcourfe I asked of you, to morrow, God permitting, we will treat of fome other ingenious Novelties.

## THE TRANSLATOR.

I fay that the Figures, \&c. would have been more intelligible to
me, drawing the Axis Z T according to its proper Pofition, that is in the half Arch of thefe Figures.] And in this confideration I have followed the Schemes of Commandine, who being the Reftorer of the Demonftrations of thefe two laft

Propofitions, hath well confidered what Ricardo here propofeth, and therefore hath drawn the faid Axis (which in the Manufcripts that he had by him is lettered F T, and not as in that of Tartaylia Z T, according to that its proper Pofition.

## A

But becaufe thus they were in the Greek Copy, I thought not
fit to alter them although unto the better.] The Schemes of thofe Manu-

[Figure 18]
fcripts that Tartaylia had feen were more imperfect then thofe in Commandines Copies; but for variety fake, take here one of Tartaylia, it being that of the Portion of a Sphære, equall to an Hemifphære, with its Axis oblique, and its Bafe dimitted into the Liquid, and Lettered as in this Edition.

## B

Now Courteous Readers, I hope that you may, amidft the great Obfcurity of the Originall in the Demonftrations of thefe two laft Propofitions, be able from the joynt light of thefe two Famous Commentators of our more famous Author, to difcern the truth of the Doctrine affirmed, namely, That Solids of the Figure of Portions of Sphæres demitted into the Liquid with their Bafes upwards fhall ftand erectly, that is, with their Axis according to the Perpendicular drawn from the Centre of the Earth unto its Circumference: And that if the faid Portions be demitted with their Bafes oblique and touching the Liquid in one Point, they fhall not rest in that Obliquity, but fhall return to Rectitude: And that laftly, if thefe Portions be demitted with their Bafes downwards, they fhall continue erect with their Axis according to the Perpendicular aforefaid: fo that no more remains to be done, but that wefet before you the 2 Books of this our Admirable Author.

## ARCHIMEDES,

HIS TRACT
DE
INSIDENTIBUS HUMIDO,
OR,
Of the NATATION of BODIES Upon, or
Submerfion In the WATER, or other LIQUIDS.
BOOK II.

## PROP. I. THEOR. I.

If any Magnitude lighter than the Liquid be demitted into the faid Liquid, it fhall have the fame proportion in Gravity to a Liquid of equal Maffe, that the part of the Magnitude demerged hath unto the whole Magnitude.

For let any Solid Magnitude, as for inftance F A, lighter than the Liquid, be demerged in the Liquid, which let be F A: And let the part thereof immerged be A, and the part above the Liquid F , It is to be demonftrated that the Magnitude F A hath the fame proportion in Gravity to a Liquid of Equall Maffe that A hath to F A. Take any Liquid Magnitude, as fuppofe N I, of equall Maffe with F A; and let F be equall to N , and A to I: and let the Gravity of the whole Magnitude F A be B, and let that of the Magnitude N I be O, and let that of I be R. Now the

[Figure 19]

Magnitude F A hath the fame proportion unto N I that the Gravity B hath to the Gravity O R: But for
afmuch as the Magnitude F A demitted into the Liquid is lighter than
the faid Liquid, it is manifeft that a Maffe of the Liquid, I, equall to the part of the Magnitude demerged, A, hath equall Gravity
with the whole Magnitnde, F A: For this was (a) above demonftrated: But B is the Gravity of the Magnitude F A, and R of I:

Therefore B and R are equall. And becaufe that of the Magnitude FA the Gravity is B: Therefore of the Liquid Body N I the Gravity is OR. As F A is to NI, fo is B to OR, or, fo is R to O R: But as R is to OR, fo is I to NI, and A to F A: Therefore

I is to NI, as F A to NI: And as I to NI fo is (b) A to F A.
Therefore F A is to N I, as A is to F A: Which was to be demonftrated.
(a) By 5. of the
firft of this.
(b) By 11. of the
fifth of Eucl.

## PROP. II. THEOR. II.

A
$\wedge\{*\}$ The Right Portion of a Right angled Conoide, when it fhall have its Axis leffe than fefquialter ejus quæ ad Axem (or of its Semi-parameter) having any what ever proportion to the Liquid in Gravity, being demitted into the Liquid fo as that its Bafe touch not the faid Liquid, and being fet ftooping, it fhall not remain ftooping, but fhall be restored to uprightneffe. I fay that the faid Portion fhall ftand upright when the Plane that cuts it fhall be parallel unto the Surface of the Liquid.

Let there be a Portion of a Rightangled Conoid, as hath been faid; and let it lye ftooping or inclining: It is to be demonftrated that it will not fo continue but fhall be reftored to rectitude. For let it be cut through the Axis by a plane erect upon the Surface of the Liquid, and let the Section of the Portion be A PO L, the Section of a Rightangled Cone, and let the Axis
[Figure 20]
of the Portion and Diameter of the Section be N O: And let the Section of the Surface of the Liquid be I S. If now the Portion be not
erect, then neither fhall A L be Pa-
rallel to I S: Wherefore N O will
not be at Right Angles with I S.

Draw therefore $\mathrm{K} \omega$, touching the Section of the Cone I , in the Point P [that is parallel to I S: and from the Point P unto I S
draw P F parallel unto O N, $\wedge\{*\}$ which fhall be the Diameter of the Section I P O S, and the Axis of the Portion demerged in the Li-
quid. In the next place take the Centres of Gravity: $\left.\wedge^{\wedge}{ }^{〔}\right\}$ and of the Solid Magnitude A P O L, let the Centre of Gravity be R; and
of I P O S let the Centre be B: $\left.\wedge^{\{ }{ }^{*}\right\}$ and draw a Line from B to R prolonged unto $G$; which let be the Centre of Gravity of the
remaining Figure I S L A. Becaufe now that N O is Sefquialter of R O, but lefs than Sefquialter ejus qux ufque ad Axem (or of its Semi-parameter;) ^\{*\} R O fhall be leffe than quæ ufque ad Axem (or than the Semi-parameter; $\wedge^{\wedge}\{*\}$ whereupon the Angle R P $\omega$ fhall be
acute. For fince the Line qux ufque ad Axem (or Semi-parameter) is greater than R O, that Line which is drawn from the Point R, and perpendicular to $\mathrm{K} \omega$, namely RT, meeteth with the line F P without the Section, and for that caufe muft of neceffity fall between the Points P and $\omega$; Therefore if Lines be drawn through B and G, parallel unto R T, they fhall contain Right Angles with the Surface of the Liquid: $\wedge\{*\}$ and the part that is within the Li-
quid fhall move upwards according to the Perpendicular that is drawn thorow B , parallel to R T , and the part that is above the Li quid fhall move downwards according to that which is drawn thorow G; and the Solid A P O L fhall not abide in this Pofition; for that the parts towards A will move upwards, and thofe towards B downwards; Wherefore N O fhall be conftituted according to the Perpendicular.]

* Supplied by Fe-
derico Comman-
dino.
B

C

D
E

F

G

## COMMANDINE.

The Demonftration of this propofition hath been much defired; which we have (in like manner as the 8 Prop. of the firft Book) reftored according to Archimedes his own Schemes, and illustrated it with Commentaries.

The Right Portion of a Rightangled Conoid, when it fhall
have its Axis leffe than Sefquialter ejus qux ufque ad Axem (or of its Semi-parameter] In the Tranflation of Nicolo Tartaglia it is fallfyread greater then Sefquialter, and fo its rendered in the following Propofition; but it is the Right Portion of a Concid cut by a Plane at Right Angles, or erect, unto the Axis: and we fay
that Conoids are then conftituted erect when the cutting Plane, that is to fay, the Plane of the Bafe, fhall be parallel to the Surface of the Liquid.

A

Which fhall be the Diameter of the Section I P O S, and the

Axis of the Portion demerged in the Liquid.] By the 46 of the firft of the Conicks of Apollonious, or by the Corollary of the 51 of the fame.

B

[Figure 21]

And of the Solid Magnitude A P

O L, let the Centre of Gravity be R;
and of I P O S let the Centre be B.]
For the Centre of Gravity of the Portion of a Right-
angled Conoid is in its Axis, which it fo divideth
as that the part thereof terminating in the vertex,
be double to the other part terminating in the Bafe; as in our Book De Centro Gravitatis Solidorum Propo. 29. we have demonftrated. And fince the Centre of Gravity of the Portion A P O L is R, O R fhall be double to RN and therefore N O fhall be Sefquialter of O R. And for the fame reafon, B the Centre of Gravity of the Portion I P O S is in the Axis P F, fo dividing it as that $P B$ is double to $B F$;

## C

And draw a Line from B to R prolonged unto G ; which let
be the Centre of Gravity of the remaining Eigure I S L A.]

For if, the Line B R being prolonged unto G, G R hath the fame proportion to R B as the Portion of the Conoid I P O S hath to the remaining Figure that lyeth above the Surface of the Liquid, the Toine G fhall be its Centre of Gravity; by the 8 of the fecond of Archimedes de Centro Gravitatis Planorum, vel de Æquiponderantibus.

D

E

R O fhall be lefs than quæ ufque ad Axem (or than the Semiparameter.] By the 10 Propofit. of Euclids fifth Book of Elements. The Line quæ ufque ad Axem, (or the Semi-parameter) according to Archimedes, is the half of that juxta quam poffunt, quæ á Sectione ducuntur, (or of the Parameter;) as appeareth by the 4 Propofit of his Book De Conoidibus \& Shpæroidibus: and for what reafon it is fo called, we have declared in the Commentaries upon him by us publifhed.

## F

Whereupon the Angle R P $\omega$ fhall be acute.] Let the Line N O be continued out to H , that fo RH may be equall to the Semi-parameter. If now from the Point H

[Figure 22]
a Line be drawn at Right Angles to N H, it fhall meet with FP without the Section; for being drawn thorow O parallel to A L , it fhall fall without the Section, by the 17 of our first Book of Conicks; Therefore let it meet in V: and becaufe F P is parallel to the Diameter, and H V perpendicular to the fame Diameter, and R H equall to the Semi-parameter, the Line drawn from the Point R to V fhall make Right Angles with that Line which the Section toucheth in the Point P: that is with $K \omega$, as fhall anon be demonstrated: Wherefore the Perpendidulat R T falleth betwixt A and $\omega$; and the Argle R $\mathrm{P} \omega$ fhall be an Acute Angle.

Let A B C be the Section of a Rightangled Cone, or a Parabola, and its Diameter B D; and let the Line E F touch the fame in the Point G: and in the Diameter B D take the Line H K equall to the Semi-parameter: and thorow G, G L being drawn parallel to the Diameter, draw KM from the

Point K at Right Angles to B D cutting G L in M: I fay
that the Line prolonged thorow Hand Mis perpendicular to
E F, which it cutteth in N .

For from the Point G draw the Line G O at Right Angles to E F cutting the Diameter in O : and again from the fame Point draw G P perpendicular to the Diameter: and let the faid Diameter prolonged cut the Line E F in que P B fhall be equall to B Q, by the 35 of
our firft Book of Conick Sections, (a) and G

[Figure 23]

Pa Mean-proportion all betmixt Q P and PO;
(b) and therefore the Square of G P fhall be equall to the Rectangle of $O P Q$ : But it is alfo equall to the Rectangle comprehended under P B and the Line juxta quam poffunt, or the Parameter, by the 11 of our firft Book of Conicks:
(c) Therefore, look what proportion $\mathrm{Q} P$ hath to
$P B$, and the fame hath the Parameter unto P O:
But $Q$ P is double unto $P B$, for that $P B$ and $B$
Q are equall, as hath been faid: And therefore
the Parameter fhall be double to the faid P O:
and by the fame Reafon P O is equall to that which we call the Semi-parameter, that is, to K H :

But (d) P G is equall to K M, and (e) the Angle O P G to the Angle H K M; for they are both

Right Angles: And therefore O G alfo is equall to H M, and the Angle P O G unto the
[Figure 24]

Angle K H M: Therefore (f) O G and H N are parallel, and the (g) Angle H N F equall to the Angle O G F; for that G O being Perpendicular to EF, H N fhall alfo be per-
pandicnlar to the fame: Which was to be demon ftrated.
(a) By Cor. of 8 . of
6. of Euclide.
(b) By 17. of the
6.
(c) By 14. of the
6.
(d) By 33. of the
1.
(e) By 4 . of the 1 .
(f) By 28. of the
1.
(g) By 29. of th

1

And the part which is within the Liquid
doth move upwards according to the Perpendicular that is drawn thorow B parallel to R T.] The reafon why this moveth upwards, and that other downwards, along the Perpendicular Line, hath been fhewn above in the 8 of the firft Book of this; fo that we have judged it needleffe to repeat it either in this, or in the reft that follow.

## G

## THE TRANSLATOR.

In the Antient Parabola (namely that affumed in a Rightangled
Cone) the Line juxta quam Poffunt qux in Sectione ordinatim ducuntur (which I, following Mydorgius, do call the Parameter) is (a)
double to that qux ducta eft à Vertice Sectionis ufque ad Axem, or in
 for want of a better word, name the Semiparameter: but in Modern Parabola's it is greater or leffer then double. Now that throughout this Book Archimedes fpeaketh of the Parabola in a Rectangled Cone, is manifeft both by the firft words of each Propofition, \& by this that no Parabola hath its Parameter double to the Line quæ eft a Sectione ad Axem, fave that which is taken in a Rightangled Cone. And in any other Parabola, for
 miparameter would be neither proper nor true: but in this cafe it may pafs
(a) Rivalt. in Ar-
chimed. de Cunoid
\& Sphæroid. Prop.
3. Lem. 1.

## PROP. III. THEOR. III.

The Right Portion of a Rightangled Conoid, when it fhall have its Axis leffe than fefquialter of the Se -mi-parameter, the Axis having any what ever proportion to the Liquid in Gravity, being demitted into the Liquid fo as that its Bafe be wholly within the faid Liquid, and being fet inclining, it fhall not remain inclined, but fhall be fo reftored, as that its Axis do ftand upright, or according to the Perpendicular.

Let any Portion be demitted into the Liquid, as was faid; and let its Bafe be in the Liquid;

[Figure 25]
and let it be cut thorow the Axis, by a Plain erect upon the Surface of the Liquid, and let the Section be A P O L, the Section of a Right angled Cone: and let the Axis of the Portion and Diameter of the

Section of the Portion be A P O L, the Section of a Rightangled Cone; and let the Axis of the Portion and Diameter of the Section be N O, and the Section of the Surface of the Liquid I S. If now the Portion be not erect, then N O fhall not be at equall Angles with I S. Draw R $\omega$ touching the Section of the Rightangled Conoid in P, and parallel to I S: and from the Point P and parall to O N draw P F: and take the Centers of Gravity; and of the Solid A P O L let the Centre be R; and of that which lyeth within the Liquid let the Centre be B; and draw a Line from B to R prolonging it to G, that G may be the Centre of Gravity of the Solid that is above the Liquid. And becaufe NO is fefquialter of R O , and is greater than fefquialter of the Semi-Parameter; it is ma-
nifeft that (a) R O is greater than the

[Figure 26]

Semi-parameter. $\left.\wedge{ }^{*}\right\}$ Let therefore R
$H$ be equall to the Semi-Parameter,
$\wedge^{\wedge}\left\{^{*}\right\}$ and O H double to H M. Forafmuch therefore as NO is fefquialter
of R O, and M O of O H, (b) the Remainder N M fhall be fefquialter of the Remainder R H: Therefore the Axis is greater than fefquialter of the Semi parameter by the quantity of the Line M O. And let it be
fuppofed that the Portion hath not leffe proportion in Gravity unto the Liquid of equall Maffe, than the Square that is made of the Exceffe by which the Axis is greater than fefquialter of the Semiparameter hath to the Square made of the Axis: It is therefore manifeft that the Portion hath not leffe proportion in Gravity to the Liquid than the Square of the Line M O hath to the Square of N O: But look what proportion the Portion hath to the Liquid in Gravity, the fame hath the Portion fubmerged to the whole Solid: for this hath been demonftrated (c) above: $\left.\wedge^{〔}{ }^{*}\right\}$ And look what proportion the fubmerged Portion hath to the whole Portion, the fame hath the Square of P F unto the Square of N O: For it hath been demonftrated in (d) Lib. de Conoidibus, that if from a Right-
angled Conoid two Portions be cut by Planes in any fafhion produced, thefe Portions fhall have the fame Proportion to each other as the Squares of their Axes: The Square of P F, therefore, hath not leffe proportion to the Square of N O than the Square of M O hath to the Square of N O: $\wedge^{\wedge}\{ \}$ Wherefore P F is not leffe than M O, $\left.\wedge^{\{ } *^{*}\right\}$ nor B P than H O. $\wedge\{*\}$ If therefore, a Right Line be drawn from H at Right Angles unto N O, it fhall meet with B P, and fhall fall betwixt B and P; let it fall in T: (e) And becaufe P F is
parallel to the Diameter, and H T is perpendicular unto the fame Diameter, and R H equall to the Semi-parameter; a Line drawn from R to T and prolonged, maketh Right Angles with the Line
contingent unto the Section in the Point P: Wherefore it alfo maketh Right Angles with the Surface of the Liquid: and that part of the Conoidall Solid which is within the Liquid fhall move upwards according to the Perpendicular drawn thorow B parallel to R T ; and that part which is above the Liquid fhall move downwards according to that drawn thorow $G$, parallel to the faid R T : And thus it fhall continue to do fo long untill that the Conoid be reftored to uprightneffe, or to ftand according to the Perpendicular.
(a) By 10. of the fifth.

A

B
(b) By 19. of the fifth.

C
(c) By 1. of this
fecond Book.
(d) By 6. De Conoilibus \& Sphæ-
roidibus of Archimedes.

D

E

F
(e) By 2. of this
fecond Book.

## COMMANDINE

## A

Let therefore R H be equall to the Semi-parameter.] So it is to be read, and not R M, as Tartaglia's Tranflation hath is; which may be made appear from that which followeth.

B

And O H double to H M.] In the Tranflation aforenamed it is fallly rendered, O N double to R M.

## C

And look what proportion the Submerged Portion hath to the whole
Portion, the fame hath the Square of P F unto the Square of N O.]
This place we have reftored in our Tranflation, at the requeft of fome friends: But it is demonftrated by Archimedes in Libro de Conoidibus \& Sphæroidibus, Propo. 26.

D
Wherefore P F is not leffe than M O.] For by 10 of the fifth it followeth that the Square of P F is not leffe than the Square of M O : and therefore neither fhall the Line P F be leße than the Line M O, by 22 of the

[Figure 27]
fixth.
E
(a) By 14. of the
fixth.
Nor B P than H O,] For as P F is to
P B, fo is M O to H O: and, by Permutation, as

PF is to MO, fo is B P to H O; But PF is not
leffe than M O as hath bin proved; (a) Therefore neither fhall B P be leffe than H O .

## F

If therefore a Right Line be drawn
from H at Right Angles unto N O, it
fhall meet with B P, and fhall fall be-
twixt B and P.] This Place was corrupt in the
Tranflation of Tartaglia: But it is thus demonstra-
ted. In regard that P F is not leffe than O M, nor P B than O H, if we fuppofe P F equall to O M, P B fhall be likewife equall to O H: Wherefore the Line drawn thorow O, parallel to AL fhall fall without the Section, by 17 of the firft of our Treatife of Conicks; And in regard that B P prolonged doth meet it beneath P; Therefore the Perpendicular drawn thorow H doth alfo meet with the fame beneath B , and it doth of neceffity fall betwixt B and P : But the fame is much more to follow, if we fuppofe P F to be greater than O M.

The Right Portion of a Right-Angled Conoid lighter than the Liquid, when it fhall have its Axis greater than Sefquialter of the Semi-parameter, if it have not greater proportion in Gravity to the Liquid [of equal Mafs] than the Exceffe by which the Square made of the Axis is greater than the Square made of the Exceffe by which the Axis is greater than fefquialter of the Semi-Parameter hath to the Square made of the Axis being demitted into the $\mathrm{Li}-$ quid, fo as that its Bafe be wholly within the Liquid, and being fet inclining, it fhall not remain fo inclined, but fhall turn about till that its Axis fhall be according to the Perpendicular.

For let any Portion be demitted into the Liquid, as hath been faid; and let its Bafe be wholly within the Liquid, And being cut thorow its Axis by a Plain erect upon the Surface of the Liquid; its Section fhall be the Section

[Figure 28]
of a Rightangled Cone: Let it be
A P O L, and let the Axis of the Portion and Diameter of the Section be N O; and the Section of the Surface of the Liquid I S. And becaufe the Axis is not according to the Perpendicular, N O will not be at equall angles with I S. Draw K $\omega$ touching the Section A P O L in P, and parallel unto I S: and thorow P, draw P F parallel unto N O: and take the Centres of Gravity; and of the Solid A P O L let the Centre be R ; and of that which lyeth above the Liquid let the Centre be B; and draw a Line from $B$ to $R$, prolonging it to $G$; which let be the Centre of Gravity of the Solid demerged within the Liquid: and moreover, take R H equall to the Semi-parameter, and let O H be double to H M ; and do in the reft as hath been faid (a) above.

Now forafmuch as it was fuppofed that the Portion hath not greater proportion in Gravity to the Liquid, than the Exceffe by which the Square N O is greater than the Square M O, hath to the faid Square N O: And in regard that whatever proportion in Gravity
the Portion hath to the Liquid of equall Maffe, the fame hath the Magnitude of the Portion fubmerged unto the whole Portion; as hath been demonftrated in the firft Propofition; The Magnitude fubmerged, therefore, fhall not have greater proportion to the
whole (b) Portion, than that which hath been mentioned: ^\{*\}And therefore the whole Portion hath not greater proportion unto that
which is above the Liquid, than the Square N O hath to the Square

M O: But the (c) whole Portion hath the fame proportion unto that which is above the Liquid that the Square N O hath to the Square P F: Therefore the Square N O hath not greater propor-
tion unto the Square P F, than it hath unto the Square M O: $\left.\wedge^{\{ }{ }^{*}\right\}$ And hence it followeth that P F is not leffe than O M, nor P B than O

H: $\wedge^{\{ }\left\{{ }^{*}\right\}$ A Line, therefore, drawn from H at Right Angles unto NO fhall meet with B P betwixt P and B: Let it be in T: And becaufe that in the Section of the Rectangled Cone P F is parallel unto the Diameter N O; and H T perpendicular unto the faid Diameter; and R H equall to the Semi-parameter: It is manifeft that R T prolonged doth make Right Angles with K P $\omega$ : And therefore doth alfo make Right Angles with I S: Therefore R T is perpendicular unto the Surface of the Liquid; And if thorow the Points B and G Lines be drawn parallel unto R T, they fhall be perpendicular unto the Liquids Surface. The Portion, therefore, which is above the Liquid fhall move downwards in the Liquid according to the Perpendicular drawn thorow B; and that part which is within the Liquid fhall move upwards according to the Perpendicular drawn thorow G; and the Solid Portion A P O L fhall not continue fo inclined, [as it was at its demerfion], but fhall move within the Liquid untill fuch time that NO do ftand according to the Perpendicular.
(a) In 4. Prop. of this.
(a) By 11. of the fifth.

A
(b) By 26. of the Book De Conoid.
\& Sphæroid.

## C

## COMMANDINE.

## A

And therefore the whole Portion hath not greater proportion unto that which is above the Liquid, than the Square N O hath to the Square M O.] For in regard that the Magnitude of the Portion demerged within the Liquid hath not greater proportion unto the whole Portion than the Exceffe by which the Square NO is greater than the Square M O hath to the faid Square N O; Converting of the Proportion, by the 26. of the fifth of Euclid, of Campanus his Tranflation, the whole Portion fhall not have leffer proportion unto the Magnitude fubmerged, than the Square N O hath unto the Exceffe by which N O is greater than the Square M O. Let a Portion be taken; and let that part of it which is above the Liquid be the firft Magnitude; the part of it which is fubmerged the fecond: and let the third Magnitude be the Square M O; and let the Exceffe by which the Square N O is greater than the Square M O be the fourth. Now of thefe Magnitudes, the proportion of the firft and fecond, unto the fecond, is not leffe than that of the third \& fourth unto the fourth: For the Square M O together with the Exceffe by which the Square N O exceedeth the Square M O is equall unto the faid Square N O: Wherefore, by Converfion of Proportion, by 30 of the faid fifth Book, the proportion of the firft and fecond unto the firft, fhall not be greater than that of the third and fourth unto the third: And, for the fame
the proportion of the whole Portion unto that part thereof which is above the Liquid fhall not be greater than that of the Square N O unto the Square M O: Which was to be demonftrated.

And hence it followeth that P F is not leffe than O M, nor P B
than O H.] This followeth by the 10 and 14 of the fifth, and by the 22 of the fixth of Euclid, as hath been faid above.

## B

A Line, therefore, drawn from Hat Right Angles unto N O fhall
meet with P B betwixt P and B.] Why this fo falleth out, we will fhew in the next.

## C

## PROP. VI. THEOR. VI.

The Right Portion of a Rightangled Conoid lighter than the Liquid, when it fhall have its Axis greater than fefquialter of the Semi-parameter, but leffe than to be unto the Semi-parameter in proportion as fifteen to fower, being demitted into the Liquid fo as that its Bafe do touch the Liquid, it fhall never stand fo enclined as that its Bafe toucheth the Liquid in one Point only.

Let there be a Portion, as was faid; and demit it into the Liquid in fuch fafhion as that its Bafe do touch the Liquid in one only Point: It is to be demonftrated that the faid Portion
fhall not continue fo, but fhall turn about in fuch manner as that its Bafe do in no wife touch the Surface of the Liquid. For let it be cut thorow its Axis by a Plane erect

[Figure 29]
upon the Liquids Surface: and let the Section of the Superficies of the Portion be A P O L, the Section of a Rightangled Cone; and the Section of the Surface of the Liquid be A S; and the Axis of the Portion and Diameter of the Section N O:
and let it be cut in F , fo as that O
F be double to FN ; and in $\omega$ fo, as that NO may be to $\mathrm{F} \omega$ in the fame proportion as fifteen to four; and at Right Angles to N O draw $\omega$ Now becaufe N O hath greater proportion unto F $\omega$ than unto the Semi-parameter, let the Semi-parameter be equall to F B:
and draw P C parallel unto A S, and touching the Section A P O L in P; and P I parallel unto N O; and firft let P I cut $\mathrm{K} \omega$ in H. For-
afmuch, therefore, as in the Portion A P O L, contained betwixt the Right Line and the Section of the Rightangled Cone, $\mathrm{K} \omega$ is parallel to A L, and P I parallel unto the Diameter, and cut by the
faid $\mathrm{K} \omega$ in H , and A S is parallel unto the Line that toucheth in P ; It is neceffary that PI hath unto PH either the fame proportion that $\mathrm{N} \omega$ hath to $\omega \mathrm{O}$, or greater; for this hath already been demonftrated: But $\mathrm{N} \omega$ is fefquialter of $\omega \mathrm{O}$; and P I, therefore, is either Sefquialter of HP , or more than fefquialter: Wherefore

P H is to H I either double, or leffe than double. Let P T be double to T I: the Centre of Gravity of the part which is within the Liquid fhall be the Point T. Therefore draw a Line from T to F prolonging it; and let the Centre of

[Figure 30]

Gravity of the part which is above the Liquid be G: and from the Point B at Right Angles unto N O draw BR. And feeing that PI is parallel unto the Diameter N O, and BR perpendicular unto the faid Diameter, and F B equall to the Semi-parameter; It is manifeft that the Line drawn thorow the Points F and R being prolonged, maketh equall
Angles with that which toucheth the Section
A P O L in the Point P: and therefore doth alfo make Right Angles with A S, and with the Surface of the Liquid: and the Lines drawn thorow T and G parallel unto F R fhall be alfo perpendicular to the Surface of the Liquid: and of the Solid Magnitude A P O L, the part which is within the Liquid moveth upwards according to the Perpendicular drawn thorow T ; and the part which is above the Liquid moveth downwards according to that drawn thorow G:

The Solid A P O L, therefore, fhall turn about, and its Bafe fhall not in the leaft touch the Surface of the Liquid, And if P I do not cut the Line $\mathrm{K} \omega$, as in the fecond Figure, it is manifeft that the Point T, which is the Centre of Gravity of the fubmerged Portion, falleth betwixt P and I: And for the other particulars remaining, they are demonftrated like as before.

A
B

C

D

## E

## COMMANDINE.

A

It is to be demonftrated that the faid Portion fhall not continue fo, but fhall turn about in fuch manner as that its Bafe do in no wife touch the Surface of the Liquid.] Thefe words are added by us, as having been omitted by Tartaglia.

Now becaufe N O hath greater proportion to $\mathrm{F} \omega$ than unto
the Semi parameter.] For the Diameter of the Portion N O hath unto F $\omega$ the fame proportion as fifteen to fower: But it was fuppofed to have leffe proportion unto the Semi-parameter than fifteen to fower: Wherefore N O hath greater proportion unto F $\omega$ than unto the Semi-parameter: And therefore (a) the Semi-parameter fhall be greater
than the faid $\mathrm{F} \omega$.

B
(a) By 10. of the fifth.

Forafmuch, therefore, as in the Portion A P O L, contained, be-
twixt the Right Line and the Section of the Rightangled Cone K
$\omega$ is parallel to A L, and P I parallel unto the Diameter, and cut by
the faid $\mathrm{K} \omega$ in H , and A S is parallel unto the Line that toucheth in P; It is neceffary that P I hath unto P H either the fame proportion that $\mathrm{N} \omega$ hath to $\omega \mathrm{O}$, or greater; for this hath already been demonftrated.] Where this is demonftrated either by Archimedes himfelf, or by any other, doth not appear; touching which we will here infert a Demonftration, after that we have explained fome things that pertaine thereto.

## C

## LEMMA I.

Let the Lines A B and A C contain the Angle B A C; and from the point D, taken in the Line A C, draw D E and D F at pleafure unto A B: and in the fame Line any Points G and L being taken, draw G H \& L M parallel to D E, \& G K and L N parallel unto F D: Then from the Points D \& G as farre as to the Line ML draw D O P, cutting G H in O, and G Q parallel unto B A. I fay that the Lines that lye betwixt the Parallels unto F D have unto thofe that lye betwixt the Parallels unto D E (namely K N to G Q or to O P; F K to D O; and $\mathrm{F} N$ to $\mathrm{D} P$ ) the fame mutuall proportion: that is to fay, the fame that A F hath to A E.

For in regard that the Triangles A F D, A K G, and A N L

[Figure 31]
are alike, and E F D, H K G, and M N L are alfo alike: There-
fore, (a) as A F is to F D, fo fhall A K be to K G; and as F D is to F E, fo fhall K G be to K H: Wherefore, ex equali, as A F is to F E, fo fhall A K be to K H: And, by Converfion of proportion, as A $F$ is to A E, fo fhall A $K$ be to $K$. It is in the fame manner proved that, as A F is to A E, fo fhall A N be to A M. Now A

N being to A M, as A K is to A H; The (b) Remainder K N fhall be unto the Remainder $\mathrm{H} M$, that is unto GQ , or unto OP , as AN is to AM; that is, as AF is to A E: Again, A K is to A H, as A F is to A E; Therefore the Remainder F K fhall be to the Remainder E H, namely to D O, as A F is to A E. We might in like manner demonstrate that fo is F N to D P: Which is that that was required to be demonstrated.
(a) By 4. of the fixth.
(b) By 5. of the fifth.

## LEMMA II.

In the fame Line A B let there be two Points R and S , fo difpofed, that A S may have the fame Proportion to A R that A F hath to A E; and thorow R draw R T parallel to E D, and thorow S draw S T parallel to FD , fo, as that it may meet with R T in the Point T. I fay that the Point T falleth in the Line A C.
[Figure 32]

For if it be poffible, let it fall fhort of it: and let R T be prolonged as farre as to AC in V : and then thorow V draw V X parallel to F D. Now, by the thing we have last demonftrated, A X fhall have the fame proportion unto AR, as A F hath to A E. But A S hath alfo the fame proportion to A R: Wherefore (a)

A $S$ is equall to $A X$, the part to the whole, which is impoffible. The fame abfurdity will follow if we fuppofe the Toint T to fall beyond the Line A C: It is therefore neceffary that it do fall in the faid A C. Which we propounded to be demonstrated.
(a) By 9. of the
fifth.

## LEMMA III.

Let there be a Parabola, whofe Diameter
let be A B; and let the Right Lines A C and B D be $\wedge\left\{{ }^{*}\right\}$ contingent to it, A C in the Point C, and B D in B: And two Lines being drawn thorow $C$, the one C E, parallel unto the Diameter; the other C F, parallel to B D; take any Point in the Diameter, as G; and as F B is to B G, fo let B G be to B H: and thorow G and H draw G K L, and H E M, parallel unto B D; and thorow M draw M N O parallel to A C, and cutting the Diameter in O: and the Line N P being drawn thorow N unto the Diameter let it be parallel to B D. I fay that H O is double to G B.

* Or touch it.

For the Line M N O cutteth the Diameter either in G, or in other Points: and if it do cut it in G, one and the fame Point fhall be noted by the two letters G and O. Therfore F C, P N, and HEM being Parallels, and AC being Parallels to M N O, they fhall make the

[Figure 33]

Triangles A F C, O P N and O H M like to
each other: Wherefore (a) O H fhall be to

H M, as A F to FC: and $\wedge^{\wedge}\left\{{ }^{*}\right\}$ Permutando,

O H fhall be to A F, as H M to F C: But the Square H M is to the Square GL as the Line H B is to the Line B G, by 20. of our firft Book of Conicks; and the Square GL is unto the Square F C, as the Line G B is to the Line B F: and the Lines H B, B G and B F are thereupon

Proportionals: Therefore the (b) Squares H M, G L and F C and there Sides, fhall alfo be Proportionals: And, therefore, as the (c) Square HM is to the Square G L, fo is the Line

H M to the Line F C: But as H M is to F C, fo is OH to AF ; and as the Square H M is to the Square G L, fo is the Line H B to B G; that is, B G to B F: From whence it followeth that O H is to A F, as B G to B F: And Permutando, O H is to B G, as A F to F B; But A F is double to F B: Therefore A B and B F are equall, by 35. of our firft Book of Conicks: And therefore N O is double to G B: Which was to be demonftrated.
(a) By 4. of the
fixth.

* Or permitting.
(b) By 22. of the fixth.
(c) By Cor. of 20 . of the fixth.


## LEMMA IV.

The fame things affumed again, and $\mathrm{M} Q$ being drawn from the Point M unto the Diameter, let it touch the Section in the Point M. I fay that H Q hath to $\mathrm{Q} O$, the fame proportion that GH hath to CN .

For make $\mathrm{H} R$ equall to GF ; and feeing that

[Figure 34]
the Triangles A F C and O P N are alike, and $P \mathrm{~N}$ equall to FC , we might in like manner demonftrate PO and FA to be equall to each other: Wherefore P O fhall be double to F B: But H O is double to G B: Therefore the Remainder P H is alfo double to the Remainder F G; that is, to R H: And therefore is followeth that P R, R H and F G are equall to one another; as alfo that $R G$ and $P F$ are equall: For $P G$ is common to both R P and G F. Since therefore, that H B is to B G, as G B is to B F, by Converfion of Proportion, $\mathrm{B} H$ fhall be to $\mathrm{H} G$, as $B \mathrm{G}$ is to GF : But Q H is to HB, as H O to B G. For by 35 of our firft Book of Conicks, in regard that Q $M$ toucheth the Section in the Point M, H B and $B \mathrm{Q}$ fhall be equall, and $\mathrm{Q} H$ double to $H B$ : Therefore, ex æquali, Q H fhall be to H G, as H O to G F; that is, to H R: and, Permutando, Q H fhall be to H O , as H G to HR : again, by Converfion, H Q fhall be to Q O, as H G to G R; that is, to P F; and, by the fame reafon, to C N: Whichwas to be demonftrated.

Thefe things therefore being explained, we come now to that
which was propounded. I fay, therefore, firft that $\mathrm{N} C$ hath to C K the fame proportion that H G hath to G B.

For fince that H Q is to Q O , as H G to CN ;
[Figure 35]
that is, to A O, equall to the faid C N: The Remainder G Q fhall be to the Remainder $\mathrm{Q} A$, as H Q to Q O: and, for the fame caufe, the Lines A C and G L prolonged, by the things that wee have above demonstrated, fhall interfect or meet in the Line Q M. Again, G Q is to $\mathrm{Q} A$, as H Q to Q O: that is, as H G to F P; as
(a) was bnt now demonstrated, But unto (b) G

Q two Lines taken together, $Q B$ that is $H B$, and $B G$ are equall: and to $Q A H F$ is equall; for if from the equall Magnitudes HB and $\mathrm{B} Q$ there be taken the equall Magnitudes F B and B A, the Re mainder fhall be equall; Therefore taking H G from the two Lines HB and B G, there fhall remain a Magnitude double to B G; that is, O H: and PF taken from FH, the Remainder is H P: Wherefore (c) O H is to H P, as G Q to Q A:

But as GQ is to QA , fo is HQ to Q O ;
that is, H G to N C: and as (d) O H is to H P, fo is G B to C K; For O H is double to G B, and H P alfo double to G F; that is, to C K; Therefore H G hath the fame proportion to N C, that G B hath to C K: And Permutando, N C hath to C K the fame proportion that H G hath to GB .
(a) By 2 . Lemma.
(b) By 4. Lemma.
(b) By 19. of the fifth.
(d) By 15. of the fifth.

Then take fome other Point at pleafure in the Section, which let be S : and thorow S draw two Lines, the one S T parallel to D B, and cutting the Diameter in the Point T; the other S V parallel to A C, and cutting C E in V. I fay that $V$ C hath greater proportion to C K , than T G hath to GB.

For prolong V S unto the Line Q M in X ; and from the Point X draw $\mathrm{X} Y$ unto the Diameter parallel to B D: G T fhall be leffe than G Y, in regard that V S is leße than V X: And, by the firft Lemma, Y G fhall be to V C, as H G to N C; that is, as G B to C K, which was demonftrated but now: And, Permutando, Y G fhall be to G B, as V C to C K: But T G, for that it is leffe than Y G, hath leffe proportion to G B, than Y G hath to the fame; Therefore V C hath greater proportion to C K. than T G hath to G B: Which was to be demonftrated. Therefore a Pofition given G K, there fhall be in the Section one only Point, to wit M, from which two Lines M E H and M N O being drawn, N C fhall have the fame proportion to C K, that H G hath to G B; For if they be drawn from any other, that which falleth betwixt A C, and the Line parallel unto it fhall alwayes have greater proportion to C K , than that which falleth betwixt G K and the Line parallel unto it hath to GB . That, therefore, is manifeft which was affirmed by Archimedes, to wit, that the Line P I hath unto P H, either the fame proportion that $\mathrm{N} \omega$ hath to $\omega \mathrm{O}$, or greater.

D

Wherefore P H is to H I either double, or leffe than double.]
If leffe than double, let P T be double to T I: The Centre of Gravity of that part of the Portion that is within the Liquid fhall be the
[Figure 36]
be the Centre of Gravity; And draw H F, and prolong it unto the Centre of that part of the Portion which is above the Liquid, namely, unto G, and the reft is demonftrated as before. And the fame is to be underftood in the Propofition that followeth.

The Solid A P O L, therefore, fhall turn about, and its Bafe fhall not in the leaft touch the Surface
of the Liquid.] In Tartaglia's Tranflation it is rendered ut Bafis ipfius non tangent fuperficiem humidi fecundum unum fignum; but we have chofen to read ut Bafis ipfius nullo modo humidi fuperficiem contingent, both here, and in the following Propofitions,
 eft; oừữ $\bar{\varepsilon}$ gos à nullo, and fo of others of the like nature.

The Right Portion of a Rightangled Conoid lighter than the Liquid, when it fhall have its Axis greater than Sefquialter of the Semi-parameter, but leffe than to be unto the faid Semi-parameter in proportion as fifteen to fower, being demitted into the Liquid fo as that its Bafe be wholly within the Liquid, it fhall never ftand fo as that its Bafe do touch the Surface of the Liquid, but fo, that it be wholly within the Liquid, and fhall not in the leaft touch its Surface.

Let there be a Portion as hath been faid; and let it be demitted into the Liquid, as we have fuppofed, fo as that its Bafe do touch the Surface in one Point only: It is to be demonftrated that the fame fhall not fo

[Figure 37]
continue, but fhall turn about in
fuch manner as that its Bafe do in no wife touch the Surface of the Liquid. For let it be cut thorow its Axis by a Plane erect upon the Liquids Surface: and let the Section be A P O L, the Section of a Rightangled
Cone; the Section of the Liquids
Surface S L; and the Axis of the
Portion and Diameter of the Section P F: and let P F be cut in R , fo, as that R P may be double to R F, and in $\omega$ fo as that P F may be to $\mathrm{R} \omega$ as fifteen to fower: and draw $\omega \mathrm{K}$ at Right Angles
to P F: (a) R $\omega$ fhall be leffe than the Semi-parameter. Therefore let R H be fuppofed equall to the Semi-parameter: and draw C O touching the Section in O and parallel unto S L ; and let N O be parallel unto P F; and firft let N O cut K $\omega$ in the Point I, as in the former Schemes: It fhall be demonftrated that N O is to O I either fefquialter, or greater than fefquialter. Let O I be leffe than double to I N ; and let O B be double to B N : and let them be difpofed like as before. We might likewife demonftrate that if a Line be drawn thorow R and T it will make Right Angles with the Line C O, and with the Surface of the Liquid: Where-
fore Lines being drawn from the Points B and G parallels unto
R T, they alfo fhall be Perpendiculars to the Surface of the Liquid:
The Portion therefore which is above the Liquid fhall move
[Figure 38]
wards according to that fame Perpendicular which paffeth thorow B; and the Portion which is within the Liquid fhall move upwards acording to that paffing thorow G: From whence it is manifeft that the Solid fhall turn about in fuch manner, as that its Bafe fhall in no wife touch the Surface of the Liquid; for that now when it toucheth but in one Point only, it moveth downwards on the part towards L. And though N O fhould not cut $\omega \mathrm{K}$, yet fhall the fame hold true.
(a) By 10 of the fifth.

## PROP. VIII. THE OR. VIII.

The Right Portion of a Rightangled Conoid, when it fhall have its Axis greater than fefquialter of the Se -mi-parameter, but leffe than to be unto the faid Semiparameter, in proportion as fifteen to fower, if it have a leffer proportion in Gravity to the Liquid, than the Square made of the Exceffe by which the Axis is greater than Sefquialter of the Semi-parameter hath to the Square made of the Axis, being demitted into the Liquid, fo as that its Bafe touch not the Liquid, it fhall neither return to Perpendicularity, nor continue inclined, fave only when the Axis makes an Angle with the Surface of the Liquid, equall to that which we fhall prefently fpeak of.

Let there be a Portion as hath been faid; and let B D be equall to the Axis: and let B K be double to K D ; and R K equall
to the Semi-parameter: and let C B be Sefquialter of B R: C D fhall be alfo Sefquialter of K R. And as the Portion is to the Liquid in Gravity, fo let the Square F Q be to the Square D B; and let F be double to Q : It is manifeft, therefore, that F Q hath to D B, lefs proportion than C B hath to B D; For C B is the Excefs by which the Axis is greater than Sefquialter of the Semi-
fame reafon, F is lefs than B R. Let $\mathrm{R} \psi$ be equall to F ; and draw $\psi$ E perpendicular to B D; which let be in power or contence the half of that which the Lines K R and $\psi \mathrm{B}$ containeth; and draw a Line from B to E: It is to be demonftrated, that the

Portion demitted into the Liquid, like as hath been faid, fhall ftand enclined fo as that its Axis do make an Angle with the Surface of the Liquid equall unto the Angle E B $\Psi$. For demit any Portion into the Liquid fo as that its Bafe

[Figure 39]
touch not the Liquids Surface; and, if it can be done, let the Axis not make an Angle with the Liquids Surface equall to the Angle E B $\Psi$; but firft, let it be greater: and the Portion being cut thorow the Axis by a Plane erect unto [or upon] the Surface of the Liquid, let the Section be A P O L the Section of a Rightangled Cone; the Section of the Surface of the Liquid X S; and let the Axis of the Portion and Diameter of the Section be N O: and draw P Y parallel to X S, and touching the Section A P O L in P; and P M parallel to N O; and P I perpendicular to N O: and moreover, let B R be equall to $\mathrm{O} \omega$, and R K to $\mathrm{T} \omega$; and let $\omega \mathrm{H}$ be perpendicular to the Axis. Now becaufe it hath been fuppofed
that the Axis of the Portion doth make an Angle with the Surface of the Liquid greater than the Angle B, the Angle P Y I fhall be greater than the Angle B: Therefore the Square P I hath greater proportion to the Square Y I, than the Square E $\Psi$ hath to the Square $\Psi$ B: But as the Square P I is to the Square Y I, fo is the

Line K R unto the Line I Y; and as the Square E $\Psi$ is to the Square
$\Psi \mathrm{B}$, fo is half of the Line K R unto the Line $\Psi \mathrm{B}$ : Wherefore (a) K R hath greater proportion to I Y , than the half of K R hath
to $\Psi$ B: And, confequently, I Y isleffe than the double of $\Psi$ B, and is the double of O I: Therefore O I is leffe than $\Psi$ B; and I $\omega$ greater than $\Psi \mathrm{R}$ : but $\Psi \mathrm{R}$ is equall to F : Therefore $\mathrm{I} \omega$ is greater
than F. And becaufe that the Portion is fuppofed to be in Gravity unto the Liquid, as the Square F Q is to the Square B D; and fince that as the Portion is to the Liquid in Gravity, fo is the part
thereof fubmerged unto the whole Portion; and in regard that as the part thereof fubmerged is to the whole, fo is the Square P M to the Square O N; It followeth, that the Square P M is to the Square N O, as the Square F Q is to the Square B D: And therefore F

Q is equall to $P$ M: But it hath been demonftrated that PH is
greater than F: It is manifeft, therefore, that $\mathrm{P} M$ is leffe than fefquialter of PH : And confequently that PH is greater than the double of H M. Let P Z be double to Z M : T fhall be the Centre of Gravity of the whole Solid; the Centre of that part of it which is within the Liquid, the Point Z ; and of the remaining
part the Centre fhall be in the Line Z T prolonged unto G. In
the fame manner we might demon-

[Figure 40]
ftrate the Line T H to be perpendicular unto the Surface of the Liquid: and that the Portion demerged within the Liquid moveth or afcendeth out of the Liquid according to the Perpendicular that fhall be drawn thorow Z unto the Surface of the Liquid; and that the part that is above the Liquid defcendeth into the Liquid according to that drawn thorow G: therefore the Portion will not continue fo inclined as was fuppofed: But neither fhall it return to Rectitude or Perpendicularity; For that of the Perpendiculars drawn thorow Z and G , that paffing thorow Z doth fall on thofe parts which are towards L ; and that that paffeth thorow G on thofe towards A : Wherefore it followeth that the Centre Z do move upwards, and $G$ downwards: Therefore the parts of the whole Solid which are towards A fhall move downwards, and thofe towards L upwards. Again let the Propofition run in other termes; and let the Axis of the Portion make an Angle with the Surface of the

Liquid leffe than that which is at B. Therefore the Square P I hath leffer Proportion unto the Square

[Figure 41]

I Y, than the Square $\mathrm{E} \Psi$ hath to the Square $\Psi$ B: Wherefore K R hath leffer proportion to I Y, than the half of $K$ R hath to $\Psi B$ : And, for the fame reafon, I $Y$ is greater than double of $\Psi$ B: but it is double of O I: Therefore O I fhall be greater than $\Psi$ B: But the Totall $\mathrm{O} \omega$ is equall to R B, and the Remainder $\omega$ I leffe than $\psi \mathrm{R}$ : Wherefore P H fhall alfo
be leffe than F . And, in regard that
$M P$ is equall to $F Q$, it is manifeft that $P M$ is greater than fefquialter of $P \mathrm{H}$; and that $\mathrm{P} H$ is leffe than double of $H \mathrm{M}$. Let PZ be double to Z M. The Centre of Gravity of the whole Solid fhall again be T ; that of the part which is within the Liquid Z ; and drawing a Line from Z to T , the Centre of Gravity of that which is above the Liquid fhall be found in that Line portracted, that is in G : Therefore, Perpendiculars being drawn thorow Z and G
unto the Surface of the Liquid that are parallel to T H, it followeth that the faid Portion fhall not ftay, but fhall turn about till that its Axis do make an Angle with the Waters Surface greater than that which it now maketh. And becaufe that when before we
did fuppofe that it made an Angle greater than the Angle B, the Poriton did not reft then neither; It is manifeft that it fhall ftay
or reft when it fhall make an Angle eqnall to B. For fo fhall I O be equall to $\Psi \mathrm{B}$; and $\omega \mathrm{I}$ equall to

[Figure 42]
$\Psi \mathrm{R}$; and P H equall to F : There-
fore M P fhall be fefquialter of PH , and P H double of H M: And therefore fince H is the Centre of Gravity of that part of it which is within the Liquid, it fhall move upwards along the fame Perpendicular according to which the whole Portion moveth; and along the fame alfo fhall the part which is above move downwards:
The Portion therefore fhall reft; forafmuch as the parts are not repulfed by each other.

A

B

C

D
E

F
G
(a) By 13. of the
fifth.
H

K
L

M

N

P

Q

## COMMANDINE.

And let C B be fefquialter of B R: C D fhall alfo be fefquialter
of K R.] In the Tranflation it is read thus: Sit autem \& CB quidem hemeolia ipfius B R: C D autem ipfius $K R$. But we at the reading of this paffage have thought fit thus to correctit; for it is not fuppofed fo to be, but from the things fuppofed is proved to be fo. For if $\mathrm{B} \psi$ be double of $\psi \mathrm{D}$, D B fhall be fefquialter of $\mathrm{B} \psi$. And becaufe E B is fefquialter of $B R$, it followeth that the (a) Remainder C D is fefquialter of $\psi R$; that is, of the Semi-parameter: Wherefore B C fhall be the Exceffe by which the Axis is greater than fefquialter of the Semi-parameter.

A
(a) By 19. of the
fifth.

And therefore F Q is leffe than B C.] For in regard that the Portion hath
the fame proportion in Gravity unto the Liquid, as the Square F Q hath to the Square D B; and hath leffer proportion than the Square made of the Exceffe by which the Axis is greater than Sefquialter of the Semi parameter, hath to the Square made of the Axis; that is, leßer than the Square C B hath to the Square B D; for the Line B D was fuppofed to be equall unto the Axis: Therefore the Square F Q fhall have to the Square D B leffer proportion than the Sqnare C B to the fame Square B D: And therefore the Square (b) F Q fhall be leße than the Square C B: And, for that reafon, the Line F Q fhall be leße than B C.

B
(b) By 8 of the
fifth.
And, for the fame reafon, F is leffe than B R.] For C B being fefqui-
alter of BR , and F Q fefquialter of F : (c) F Q fhall be likewife leffe than B C; and F
leße than B R.

C
(c) By 14 of the
fifth.

Now becaufe it hath been fuppofed that the Axis of the Portion
doth make an Angle with the Surface of the Liquid greater than the Angle B, the Angle P Y I fhall be greater than the Angle B.]
For the Line P Y being parallel to the Surface of the Liquid, that is, to XS; (d) the Angle

P Y I fhall be equall to the Angle contained betwixt the Diameter of the Portion N O, and the Line X S: And therefore fhall be greater than the Angle B.

D
(d) By 29 of the firft.

Therefore the Square P I hath greater proportion to the Square

Y I, than the Square E $\Psi$ hath to the Square $\Psi$ B] Let the Triangles P I Y and $\mathrm{E} \psi \mathrm{B}$, be defcribed apart: And feeing that the Angle P Y I is greater than the Angle E B $\psi$, unto the Line I Y, and at the Point Y affigned in

[Figure 43]
the fame, make the Angle V Y I equall to the Angle E B $\psi$; But the Right Angle at I , is equall unto the Right Angle at $\psi$; therefore the

Remaining Angle Y V I is equall to the Remaining Angle B E $\psi$. And therefore the
(e) Line V I hath to the Line I Y the fame proportion that the Line $\mathrm{E} \psi$ hath to $\psi$ B: But the (f) Line P I, which is greater than V I, hath unto I Y greater proportion than V I hath un-
to the fame: Therefore (g) T I fhall have greater proportion unto I Y, than E $\psi$ hath to $\psi$ B: And, by the fame reafon, the Square T I fhall have greater proportion to the Square I Y, than the Square E $\psi$ hath to the Square $\psi$ B.

## E

(e) By 4. of the
fixth.
(f) By 8 . of the
fifth.
(g) By 13 of the
fifth.

## F

But as the Square PI is to the Square Y I, fo is the Line K R unto the Line I Y] For by 11. of the firft of our Conicks, the Square P I is equall to the Rectangle contained under the Line I O, and under the Parameter; which we fuppofed to be eqnall to the Semi-parameter; that is, the double of K R:

But I Y is double of I O, by 33 of the fame: And, therefore, the (h) Rectangle made of K R and I Y, is equall to the Rectangle contained under the Line I O, and under the Parameter;
that is, to the Square P I: But as the (i) Rectangle compounded of K R and I Y is to the Square I Y, fo is the Line K R unto the Line I Y: Therefore the Line K R fhall have unto I Y, the fame proportion that the Rectangle compounded of K R and I Y; that is, the Square P I hath to the Square I Y.
(h) By 26. of the
fixth.
(i) By Lem. 22 of the tenth.

## G

And as the Square $\mathrm{E} \Psi$ is to the Square $\Psi \mathrm{B}$, fo is half of the
Line K R unto the Line $\psi$ B.] For the Square $\mathrm{E} \psi$ having been fuppofed equall to half the Rectangle contained under the Line $\mathrm{K} R$ and $\psi \mathrm{B}$; that is, to that contained under the half of $K R$ and the Line $\psi B$; and feeing that as the ( $k$ ) Rectangle made of half $K R$
and of $B \psi$ is to the Square $\psi B$, fo is half $K R$ unto the Line $\psi B$; the half of $K R$ fhall have the fame proportion to $\psi \mathrm{B}$, as the Square $\mathrm{E} \psi$ hath to the Square $\psi \mathrm{B}$.
(k) By Lem. 22 of the tenth.

H
And, confequently, I Y is leffe than the double of $\psi$ B.]
For, as half $K R$ is to $\psi B$, fo is $K R$ to another Line: it fhall be (1) greater than I Y; that
is, than that to which $\mathrm{K} R$ hath leffer proportion; and it fhall be double of $\psi \mathrm{B}$ : Therefore I Y is leffe than the double of $\psi \mathrm{B}$.
(l) By 10 of the
fifth.

K

And I $\omega$ greater than $\psi$ R.] For O having been fuppofed equall to B R, if from BR, $\psi$ B be taken, and from O $\omega$, O I, which is leffer than B, be taken; the Remainder $\mathrm{I} \omega$ fhall be greater than the Remainder $\Psi$ R.

L

And, therefore, F Q is equall to P M.] By the fourteenth of the fifth of Euclids Elements: For the Line ON is equall to B D.

M
But it hath been demonftrated that PH is greater than F .]
For it was demonftrated that $I \omega$ is greater than $F$ : And $P H$ is equall to $I \omega$.

N

In the fame manner we might demonftrate the Line T H
to be Perpendicular unto the Surface of the Liquid.] For $\mathrm{T} \alpha$ is equall
to K R; that is, to the Semi-parameter: And, therefore, by the things above demonstrated, the Line T H fhall be drawn Perpendicular unto the Liquids Surface.

O

Therefore, the Square P I hath leffer proportion unto the
Square I Y, than the Square E hath to the Square $\psi$ B.]
Thefe, and other particulars of the like nature, that follow both in this and the following Propofitions, fhall be demonftrated by us no otherwife than we have done above.

Therefore Perpendiculars being drawn thorow Z and G , unto the Surface of the Liquid, that are parallel to T H, it followeth that the faid Portion fhall not ftay, but fhall turn about till that its Axis do make an Angle with the Waters Surface greater than that which it now maketh.] For in that the Line drawn thorow G, doth fall perpendicularly towards thofe parts which are next to L ; but that thorow Z , towards thofe next to A; It is necefflary that the Centre $G$ do move downwards, and $Z$ upwards: and, therefore, the parts of the Solid next to L fhall move downwards, and thofe towards A upwards, that the Axis may makea greater Angle with the Surface of the Liquid.

Q
For fo fhall I O be equall to $\psi \mathrm{B}$; and $\omega$ I equall to I R; and
P H equall to F.] This plainly appeareth in the third Figure, which is added by us.

The Right Portion of a Rightangled Conoid, when it fhall have its Axis greater than Sefquialter of the Semi-parameter, but leffer than to be unto the faid Semi-parameter in proportion as fifteen to four, and hath greater proportion in Gravity to the Liquid, than the excefs by which the Square made of the Axis is greater than the Square made of the Excefs, by which the Axis is greater than Sefquialter of the Semiparameter, hath to the Square made of the Axis, being demitted into the Liquid, fo as that its Bafe be wholly within the Liquid, and being fet inclining it fhall neither turn about, fo as that its Axis ftand according to the Perpendicular, nor remain inclined, fave only when the Axis makes an Angle with the Surface of the Liquid, equall to that aßigned as before.

Let there be a Portion as was faid; and fuppofe D B equall to the Axis of the Portion: and let B K be double to K D; and K R equall to the Semi-parameter: and C B Sefquialter of B R. And as the Portion is to the Liquid in Gravity, fo let the Exceffe by which the Square B D exceeds the Square F Q be to the Square B D: and let F be double to Q: It is manifeft, therefore, that the Exceffe by which the

[Figure 44]

Square B D is greater than the Square B C hath lefser proportion to the Square B D, than the Excefs by which the Square $\mathrm{B} D$ is greater than the Square F Q hath to the Square B D; for B C is the Excefs by which the Axis of the Portion is greater than Sefquialter of the Semi-parameter: And, therefore,
the Square B D doth more exceed the Square F Q, than doth the Square B C: And, confequently, the Line F Q is lefs than B C;
and F lefs than B R. Let R $\Psi$ be equall to F ; and draw $\Psi \mathrm{E}$ perpendicular to $B \mathrm{D}$; which let be in power the half of that which the Lines K R and $\Psi$ B containeth; and draw a Line from B to E: I fay that the Portion demitted into the Liquid, fo as that its Bafe be wholly within the Liquid, fhall fo ftand, as that its Axis do make an Angle with the Liquids Surface, equall to the Angle B. For let the Portion be demitted into the Liquid, as hath been faid; and let the Axis not make an Angle with the Liquids Surface, equall to B, but firft a greater: and the fame being cut thorow the Axis by a Plane erect unto the Surface of the Liquid, let the Section of the Portion be A P O L, the Section of a Rightangled Cone; the Section of the Surface of the Liquid $\Gamma$ I; and the Axis of the Portion and Diameter of the Section N O; which let be cut in the Points $\omega$ and T , as before: and draw Y P, parallelto $\Gamma \mathrm{I}$, and touching the Section in P , and MP parallel to N O , and P S perpendicular to the Axis. And becaufe now that the Axis of the Portion maketh an Angle with the Liquids Surface greater than the Angle B, the Angle S Y P fhall alfo be greater than the Angle B: And, therefore, the Square P S hath greater proportion to the Square

S Y, than the Square $\Psi$ E hath to the Square $\Psi$ B: And, for that caufe, $K$ R hath greater proportion to $S Y$, than the half of $K$ R hath to $\Psi \mathrm{B}$ : Therefore, S Y is lefs than the double of $\Psi \mathrm{B}$; and

S O lefs than $\psi$ B: And, therefore, $\mathrm{S} \omega$ is greater than $\mathrm{R} \psi$; and
P H greater than F. And, becaufe that the Portion hath the fame proportion in Gravity unto the Liquid, that the Excefs by which the Square B D, is greater than the Square F Q, hath unto the Square B D; and that as the Portion is in proportion to the Liquid in Gravity, fo is the part thereof fubmerged unto the whole Portion; It followeth that the part fubmerged, hath the fame proportion to the whole Portion, that the Excefs by which the Square B D is greater than the Square F Q hath unto the Square B D: And, therefore, the whole Portion fhall have the fame propor-
tion to that part which is above the

[Figure 45]

Liquid, that the Square B D hath to the Square F Q: But as the whole Portion is to that part which is above the Liquid, fo is the Square N O unto
the Square P M: Therefore, P M fhall be equall to F Q : But it
hath been demonftrated, that PH is greater than F. And, therefore, MH thall be lefs than que and P H greater than double of H M. Let therefore, P Z be double to Z M :
and drawing a Line from Z to T pro-
[Figure 46]
long it unto G. The Centre of
Gravity of the whole Portion fhall be T; of that part which is above the Liquid Z ; and of the Remainder which is within the Liquid, the Centre fhall be in the Line Z T prolonged; let it be in G: It fhall be demonftrated, as before, that T H is perpendicular to the Surface of the Liquid, and that the Lines
drawn thorow Z and G parallel to the faid T H , are alfo perpendiculars unto the fame: Therefore, the Part which is above the Liquid fhall move downwards, along that which pafseth thorow Z; and that which is within it, fhall move upwards, along that which pafseth thorow G: And, therefore, the Portion fhall not remain fo inclined, nor fhall fo turn about, as that its Axis be perpendicular
unto the Surface of the Liquid; for the parts towards L fhall move downwards, and thofe towards A upwards; as may appear by the things already demonftrated. And, if the Axis fhould make an Angle with the Surface of the Liquid, lefs than the Angle B; it fhall in like manner be demonftrated, that the Portion will not
reft, but incline untill that its Axis do make an Angle with the Surface of the Liquid, equall to the Angle B.

A

B

C

D

E

F

G

And, therefore, the Square B D doth more exceed the Square

F Q, than doth the Square B C: And, confequently, the Line F Q, is lefs than B C; and F lefs than B R.] Becaufe the Excefs by which the Square B D exceedeth the Square B C; having lefs proportion unto the Square B D, than the Excefs by which the Square B D exceedeth the Square F Q, hath to the faid Square; (a) the Excefs by which the Square B D exceedeth the Square B C fhall be lefs than the Excefs
by which it exceedeth the Square F Q: Therefore, the Square F Q is lefs than the Square B C: and, confquently, the Line F Q lefs than the Line BC: But F Q hath the fameproportion to F, that B C hath to B R; for the Antecedents are each Sefquialter of their confequents: And (b) F Q being lefs than B C, F fhall alfo be lefs than B R.

A
(a) By 8. of the
fifth.
(b) By 14. of the
fifth.

And, for that caufe, K R hath greater proportion to S Y, than the half of K R hath to $\psi$ B.] For K R is to S Y, as the Square P S is to the Square

S Y: and the half of the Line K R is to the Line $\psi B$, as the Square $E \psi$ is to the Square $\psi B$.

B

And S O lefs than $\psi$ B.] For S Y is double of S O.

## C

And P H greater than F.] For P H is equall to $\mathrm{S} \omega$, and $\mathrm{R} \psi$ equall to F .

D

And, therefore, the whole Portion fhall have the fame propor-
tion to that part which is above the Liquid, that the Square B D
hath to the Square F Q] Becaufe that the part fubmerged, being to the whole Portion as the Excefs by which the Square B D is greater than the Square F Q, is to the Square B D; the whole Portion, Converting, fhall be to the part thereof fubmerged, as the Square B D is to
the Excefs by which it exceedeth the Square F Q: And, therefore, by Converfion of Proportion, the whole Portion is to the part thereof above the Liquid, as the Square B D is to the Square, F que for the Square B D is fo much greater than the Excefs by which it exceedeth the Squar, F Q as is the faid Square F que

## E

## F

For the parts towards L fhall move downwards, and thofe towards A upwards.] We thus carrect thefe words, for in Tartaglia's Tranflation it is fallly, as I conceive, read Quoniam qux ex parte L ad fuperiora ferentur, becaufe the Line thàt paffeth thorow Z falls perpendicularly on the parts towards L , and that thorow G falleth perpendicularly on the parts towards A: Whereupon the Centre Z, together with thofe parts which are towards L fhall move downwards; and the Centre G, together with the parts which are towards A upwards.

## G

It fhall in like manner be demonftrated that the Portion fhall not reft, but incline untill that its Axis do make an Angle with the Surface of the Liquid, equall to the Angle B.] This may be eafily demonftratred, as nell from what hath been faid in the precedent Propofition, as alfo from the two latter Figures, by us inferted

## PROP. X. THEOR. X.

The Right Portion of a Rightangled Conoid, lighter than the Liquid, when it fhall have its Axis greater than to be unto the Semiparameter, in proportion as fifteen to four, being demitted into the Liquid, fo as
that its Bafe touch not the fame, it fhall fometimes
ftand perpendicular; fometimes inclined; and fometimes fo inclined, as that its Bafe touch the Surface of the Liquid in one Point only, and that in two Po-
fitions; fometimes fo that its Bafe be more fubmer-
ged in the Liquid; and fometimes fo as that it doth not in the leaft touch the Surface of the Liquid;
according to the proportion that it hath to the Liquid in Gravity. Every one of which Cafes fhall be anon demonftrated.

C

D

E

Let there be a Portion, as hath been faid; and it being cut thorow its Axis, by a Plane erect unto the Superficies of the Liquid, let the Section be A P O L, the Section of a Right angled Cone; and the Axis of the Portion and Diameter of the Section B D: and let B D be cut in the Point K, fo as that B K be double of $\mathrm{K} D$; and in C , fo as that $\mathrm{B} D$ may have the fame proportion to K C , as fifteen to four: It is manifeft, therefore, that K C is greater than the Semi-parameter: Let the
parameter be equall to K R : and

[Figure 47]
let D S be Sefquialter of $K$ R: but $S B$ is alfo Sefquialter of $B R$ :
Therefore, draw a Line from A to B; and thorow C draw C E Perpendicular to $\mathrm{B} D$, cutting the Line A B in the Point E; and thorow E draw E Z parallel unto B D. Again, A $B$ being divided into two equall parts in T, draw T H parallel to the fame B D: and let Sections of Rightangled Cones be defcribed, A E I about the Diameter E Z; and A T D about the Diameter T H; and let them be like to the

Portion A B L: Now the Section of the Cone A E I, fhall pafs
thorow K ; and the Line drawn from R perpendicular unto B D , fhall cut the faid A E I; let it cut it in the Points Y G: and thorow Y and G draw P Y Q and O G N parallels unto B D, and cutting A T D in the Points F and X: laftly, draw $\mathrm{P} \Phi$ and O X touching the Section A P O L in the Points P and O . In regard,
therefore, that the three Portions A P O L, A E I, and A T D are contained betwixt Right Lines, and the Sections of Rightangled Cones, and are right alike and unequall, touching one another, upon one and the fame Bafe; and N X G O being drawn from the Point N upwards, and Q F Y P from Q: O G fhall have to G X a proportion compounded of the proportion, that I L hath to $\mathrm{L} A$, and of the proportion that A D hath to DI: But I L is to L A, as two to five: And C B is to B D, as fix to fifteen; that is, as two
to five: And as C B is to B D, fo is E B to B A; and D Z to

D A: And of D Z and D A, L I and L A are double: and A D
is to D I, as five to one: But the proportion compounded of the proportion of two to five, and of the proportion of five to one, is
the fame with that of two to one: and two is to one, in double proportion: Therefore, O G is double of GX: and, in the fame
manner is P Y proved to be double of Y F: Therefore, fince that D S is Sefquialter of K R; B S fhall be the Excefs by which the Axis is greater than Sefquialter of the Semi-parameter. If therefore, the Portion have the fame proportion in Gravity unto the Liquid, as the Square made of the Line B S, hath to the Square made of $\mathrm{B} D$, or greater, being demitted into the Liquid, fo as hat its Bafe touch not the Liquid, it fhall ftand erect, or perpendicular: For it hath been demonftrated above, that the Portion whofe

Axis is greater than Sefquialter of the Semi-parameter, if it have not lefser proportion in Gravity unto the Liquid, than the Square
made of the Excefs by which the Axis is greater than Sefquialter of the Semi-parameter, hath to the Square made of the Axis, being demitted into the Liquid, fo as hath been faid, it fhall ftand erect, or Perpendicular.

F

G

H

K

L

M

N

O

P

Q

R

## COMMANDINE.

The particulars contained in this Tenth Propofition, are divided by Archimedes into five Parts and Conclufions, each of which he proveth by a diftinct Demonftration.

## A

It fhall fometimes ftand perpendicular.] This is the firft Conclufion, the Demonstration of which he hath fubjoyned to the Propofition.

B

And fometimes fo inclined, as that its Bafe touch the Surface of the Liquid, in one Point only.] This is demonftrated in the third Conclufion.

Sometimes, fo that its Bafe be moft fubmerged in the Liquid.]

This pertaineth unto the fourth Conclufion.

C

And, fometimes, fo as that it doth not in the leaft touch the Sur-
face of the Liquid.] This it doth hold true two wayes, one of which is explained is the fecond, and the other in the fifth Conclufion.

D

According to the proportion, that it hath to the Liquid in Gra-
vity. Every one of which Cafes fhall be anon demonftrated.]
In Tartaglia's Verfion it is rendered, to the confufion of the fence, Quam autem proportionem habeant ad humidum in Gravitate fingula horum demonftrabuntur.

## E

It is manifeft, therefore, that K C is greater than the Semi-
parameter] For, fince $\mathrm{B} D$ hath to K C the fame proportion, as fifteen to four, and hath unto the Semi-parameter greater proportion; (a) the Semi-parameter fhall be lefs than K C.

F
(a) By 10. of the fifth.

Let the Semi-parameter be equall to KR.] We have added thefe words, which are not to be found in Tartaglia.

## G

But S B is alfo Sefquialter of BR.] For, D B is fuppofed Sefquialter of

B K; and D S alfo is Sefquialter of K R: Wherefore as (b) the whole D B, is to the whole $B K$, fo is the part D S to the part K R. Therefore, the Remainder S B, is alfo to the

Remainder B R , as D B is to B K .

H
(b) By 19 of the
fifth.

And let them be like to the Portion A B L.] Apollonius thus defineth
like Portions of the Sections of a Cone, in Lib. 6. Conicornm, as Eutocius writeth $\wedge\{*\}$;


that is, In both of which an equall number of Lines being drawn parallel to the Bafe; the parallel and the Bafes have to the parts of the Diameters, cut off from
the Vertex, the fameproportion: as alfo, the parts cut off, to the parts cut off.
Now the Lines parallel to the Bafes are drawn, as I fuppofe, by making a Rectilineall Figure (cal-
 ber of Sides in both. Therefore, like Portions are cut off from like Sections of a Cone; and their Diameters, whether they be perpendicular to their Bafes, or making equall Angles with their Bafes, have the fame proportion unto their Bafes.

## K

* Upon prop. 3 lib. 2

Archim. Æqui-
pond.
Vide Archim, ante
prop. 2. lib. 2.
Æquipond.

## L

Now the Section of the Cone A E I fhall pafs thorow K.]
For, if it be poffible, let it not pafs thorow K, but thorow fome other Point of the Line D B, as thorow V. Inregard, therefore, that in the Section of the Right-angled Cone A E I, whofe Diameter is E Z, A E is drawn and prolonged; and D B parallel unto the Diameter, cutteth both A E and A I; A E in B, and A I in D; D B fhall have to B V, the fame proportion
that A Z hath to Z D; by the fourth Propofition of Archimedes, De quadratura Parabolx: But A Z is Sefquialter of Z D; for it is as three to two, as we fhallanon demon-
ftrate: Therefore D B is Sefquialter of B V; but D B and B K are Sefquialter:
And, therefore, the Lines (c) B V and B K are equall: Which is imposfible:
Therefore the Section of the Right-angled Cone A E I, fhall pafs thorow the Point K; which we would demonstrate.
(c) By 9 of the
fifth,
In regard, therefore, that the three Portions A P O L, A E I
and A T D are contained betwixt Right Lines and the Sections
of Right-angled Cones, and are Right, alike and unequall, touching one another, upon one and the fame Bafe.] After thefe words, upon one and the fame Bafe, we may fee that fomething is obliterated, that is to be defired: and for the Demonftration of thefe particulars, it is requifite in this place to premife fome things: which will alfo be neceffary unto the things that follow.

M

## LEMMA. I.

Let there be a Right Line A B; and let it be cut by two Lines, parallel to one another, A C and D E, fo, that as A B is to B D. fo A C may be to D E. I fay that the Line that conjoyneth the Points C and B fhall likewife pafs by E.

[Figure 48]

For, if poffible, let it not pafs by E, but either above or below it. Let it first pafs below it, as by F. The Triangles A B C and D B F fhall be alike: And, therefore, as (a) AB is to BD,
fo is A C to D F: But as A B is to B D, fo was A C to D E: Therefore (b) D F fhall be equall to

D E: that is, the part to the whole: Which is
abfurd. The fame abfurditie will follow, if the
Line C B be fuppofed to pafs above the Point E:
And, therefore, C B muft of necesfity pafs thorow E: Which was required to be demonftrated.
(a) By 4. of the fixth.
(b) By 9. of the fifth.

## LEMMA. II.

Let there be two like Portions, contained betwixt Right Lines, and the Sections of Right-angled Cones; A B C the greater, whofe Diameter let be B D; and E F C the lefser, whofe Diameter let be F G: and, let them be fo applyed to one another, that the greater include the lefser; and let their Bafes A C and E C be in the fame Right Line, that the fame Point C , may be the term or bound of them both: And, then in the Section A B C, take any Point, as H; and draw a Line from H to C. I fay, that the Line H C, hath to that part of it felf, that lyeth betwixt C and the Section E F C, the fame proportion that A C hath to C E.

Draw B C, which fhall pafs thorow F, For, in regard, that the Portions are alike, the Diameters with the Bafes contain equall Angles: And, therefore, B D and F G are parallel to one another: and B D is to A C, as F G it to E C: and, Permutando, B D is to F G, as A C is to C E; that is, (a) as their halfes D C to C G; therefore, it followeth, by the
preceding Lemma, that the Line B C fhall pafs by the Point F. Moreover, from the Point H unto the Diameter B D, draw the Line H K, parallel to the Bafe A C: and, draw a Line
[Figure 49]
from K to C, cutting the Diameter F G in L: and, thorow L, unto the Section E F. G, on the part E, draw the Line L M, parallel unto the fame Bafe A C. And, of the Section A B C, let the Line B N be the Parameter; and, of the Section E F C, let F O be the Parameter. And, becaufe the Triangles C B D and C F G are alike; (b) therefore, as B C is to C F, fo fhall D C be
to C G, and B D to F G. Again, becaufe the Triangles C K B and C L F, are alfo alike to one another; therefore, as B C is to C F, that is, as B D is to F G, fo fhall K C be to C L, and B K to F L: Wherefore, K C to C L, and,

B K to F L, are as D C to C G; that is, (c) as their duplicates A C and C E: But as
B D is to F G, fo is D C to C G; that is, A D to E G: And, Permutando, as B D is to A D, fo is F G to E G: But the Square A D, is equall to the Rectangle D B N, by the 11 of our firft of Conicks: Therefore, the (d) three Lines B D, A D and B N are

Proportionalls. By the fame reafon, likewife, the Square E G being equall to the Rectangle G F O, the three other Lines F G, E G and F O, fhall be alfo Proportionals: And, as B D is to A D, fo is F G to E G: And, therefore, as A D is to B N, fo is E G to F O: Ex equali, therefore, as D B is to B N, fo is GF to F O: And, Permutando, as D B is to G F, fo is B N to F O: But as D B is to G F, fo is B K to F L: Therefore, B K is to F L, as B N is to F O: And, Permutando, as B K is to B N, fo is FL to F O. Again, becaufe the (e) Square H K is equall to the Rectangle BN; and the Square ML, equall
to the Rectangle L F O, therefore, the three Lines B K, K H and B N fhall be Proportionals: and F L, L M, and F O fhall alfo be Proportionals: And, therefore, (f) as the Line

B K is to the Line B N, fo fhall the Square B K, be to the Square H K: And, as the Line F L is to the Line F O, fo fhall the Square F L be to the Square L M: Therefore, becaufe that as B K is to B N, fo is F L to F O; as the Square

B K is to the Square K H, fo fhall the Square F L be to the Square L M: Therefore, (g) as the Line B K is to the Line K H, fo is the Line F L to L M: And, Permutando, as B K is to F L, fo is K H to L M: But B K was to F L, as K C to C L: Therefore, K H is to L M, as K C to C L: And, therefore, by the preceding Lemma, it is manifeft that the Line H C alfo fhall pafs thorow the Point M: As K C, therefore, is to C L, that is, as A C to C E, fo is H C to C M; that is, to the fame part of it felf, that lyeth betwixt C and the Section E F C. And, in like manner might we demonftrate, that the fame happeneth
in other Lines, that are produced from the Point C, and the Sections E B C. And, that B C hath the fame proportion to C F, plainly appeareth; for B C is to C F, as D C to C G; that is, as their Duplicates A C to C E.
(a) By 15. of the fifth.
(b) By 4. of the fixth.
(c) By 15. of the fifth.
(d) By 17. of the fixth.
(e) By 11 of our firft of Conicks.
(f) By Cor. of 20. of the fixth.
(g) By 23. of the fixth.

From whence it is manifeft, that all Lines fo drawn, fhall be cut by the faid Section in the fame proportion. For, by Divifion and Converfion, $C M$ is to $M H$, and C F to F B, as C E to E A.

## LEMMA. III.

And, hence it may alfo be proved, that the Lines which are drawn in like Portions, fo, as that with the Bafes, they contain equall Angles, fhall alfo cut off like Portions; that is, as in the foregoing Figure, the Portions H B C and M F C, which the Lines C H and C M do cut off, are alfo alike to each other.

For let C H and C M be divided in the midst in the Points P and que and thorow thofe Points draw the Lines R P S and T Q V parallel to the Diameters. Of the Portion H S C the Diameter fhall be P S, and of the Portion M V C the Diameter fhall be

Q V. And, fuppofe that as the Square C R is to the Square C P, fo is the Line B N unto another Line; which let be S X: And, as the Square C T is to the Square C Q fo let F O be to $V$ Y. Now it is manifeft, by the things which we have demonftrated, in our Commentaries, upon the fourth Propofition of Archimedes, De Conoidibus \& Spheæroidibus, that the Square C P is equall to the Rectangle P S X; and alfo, that the Square C Q is equall to the Rectangle Q V Y; that is, the Lines S X and V Y, are the Parameters of the Sections H S C and M V C: But fince the Triangles C P R and C Q T are alike; C R fhall have to C P, the fame Proportion that C T hath to C Q: And, therefore, the (a) Square C R fhall have
to the Square C P, the fame proportion that the

[Figure 50]

Square C T hath to the Square C Q: Therefore, alfo, the Line $\mathrm{B} N$ fhall be to the Line S X , as the Line F O is to V Y: But H C was to C M, as A C to C E: And, therefore, alfo, their halves $C P$ and $C$, are alfo to one another, as A D and E G: And. Permutando, C P is to A D, as C Q is to E G:
But it hath been proved, that A D is to $B N$, as E G to F O; and B N to S X, as F O to
V Y: Therefore, exæquali, C P fhall be to $S \mathrm{X}$, as C Q is to V Y. And, fince the
Square C P is equall to the Rectangle PSX, and the Square C Q to the Rectangle Q V Y, the three Lines S P, PC and S X fhall be proportionalls, and V Q, Q C and V Y fhal be Proportionalls alfo: And therefore alfo S P fhall be to P C as V Q to Q C And as P C is to C H, fo fhall Q C. be to C M: Therefore, ex æquali, as S P the Diameter of the Portion H S C is to its Bafe C H, fo is V Q the Diameter of the portion M V S the Bafe C M; and the Angles which the Diameter with the Bafes do contain, are equall; and the Lines S P and V Q are parallel: Therefore the Portions, alfo, H S C and M V C fhall be alike: Which was propofed to be demonftrated
(a) By 22. of the
fixth.

## LEMMA. IV.

Let there be two Lines A B and C D; and let them be cut in the Points E and F, fo that as A E is to E B, C F may be to F D: and let them be cut again in two other Points $G$ and $H$; and let C H be to H D, as A G is to G B. I fay that C F fhall be to F H as A E is E G.

For in regard that as A E is to E B, fo is C F to F D; it followeth that, by Compounding,
as A B is to E B, fo fhall C D be to F D. Again, fince that as A G is to G B, fo is C H, to H D; it followeth that, by Compounding and Converting, as G B is to A B, fo fhall H D be
[Figure 51]

C D: Therefore, ex æquali, and Converting as E B is to G B, fo fhall F D be to H D; And, by Converfion of Propofition, as E B is to E G, fo fhall F D be to F H: But as A E is to E B, fo is C F to F D:
Ex æquali, therefore, as $A E$ is to $E G, f o$
fhall CF be to F H. Again, another way. Let
the Lines A B and C D be applyed to one another, fo as that they doe make an Angle at the parts A and C ;
and let A and C be in one and the fame Point: then
draw Lines from D to B , from H to G , and from F to E . And fince that as A E is to $\mathrm{E} B$, fo is C F, that is A F to F D; therefore F E fhall be parallel to D B; (a) and likewife

H G fhall be parallel to D B; for that A H is to H D, as A G to G B: (b) Therefore F E and H G are parallel to each other: And confequently, as A E is to E G, fo is A H, that is,

C F to F H: Which was to be demonftrated.
(a) By 2 . of the
fixth.
(b) By 30 of the
firf.

## LEMMA. V.

Again, let there be two like Portions, contained betwixt Right Lines and the Sections of Right-angled Cones, as in the foregoing figure, A B C, whofe Diameter is B D; and E F C, whofe Diameter is F G; and from the Point E, draw the Line E H parallel to the Diameters B D and F G; and let it cut the Section A B C in K: and from the Point C draw C H touching the Section A B C in C, and meeting with the Line E H in H; which alfo toucheth the Section E F C in the fame Point C , as fhall be demonftrated: I fay that the Line drawn from C H unto the Section E F C fo as that it be parallel to the Line E H, fhall be divided in the fame proportion by the Section A B C, in which the Line C A is divided by the Section E F C; and the part of the Line C A which is betwixt the two Sections, fhall anfwer in proportion to the part of the Line drawn, which alfo falleth betwixt the fame Sections: that is, as in the foregoing Figure, if D B be produced untill it meet with C H in L, that it may interfect the Section E F C in the Point M, the Line L B fhall have to B M the fame proportion that C E hath to E A.

For let G F be prolonged untill it meet the fame Line C H in N, cutting the Section A B C in O ; and drawing a Line from B to C , which fhall paffe by F , as hath been fhewn, the
[Figure 52]

Triangles C G F and C D B fhall be alike; as alfo the Triangles C F N and C B L: Wherefore
(a) as G F is to D B, fo fhall C F b to C B:

And as (b) C F is to C B, fo fhall F N be to B L: Therefore G F fhall be to D B, as F N
to B L: And, Permutando, G F fhall be to
F N, as D B to B L: But D B is equall to
B L, by 35 of our Firft Book of Conicks:
Therefore (c) G F alfo fhall be equall to F N:

And by 33 of the fame, the Line C H toucheth the Section E F C in the fame Point. Therefore, drawing a Line from C to M , prolong it untill it meet with the Section A B C in P; and from P unto $\mathrm{A} C$ draw P Q parallel to $\mathrm{B} D$. Becaufe, now, that the Line C H toucheth the Section E F C in the Point C; L M fhall have the fame proportion to M D that C D hath to D E, by the Fifth Propofition of Archimedes in his Book De Quadratura Patabolx: And by reafon of the Similitude of the Triangles C M D and C P Q as C M is to C D, fo fhall C P be to C Q: And, Permutando, as C M is to C P, fo fhall C D be to C Q: But as C M is to C P, fo is C E to C A,; as we have but even now demonftrated: And therefore, as C E is to C A, fo is C D to C que that is as the whole is to the whole, fo is the part to the part: The remainder, therefore, D E is to the Remainder Q A, as C E is to C A; that is, as C D is to C Q: And, Permutando, C D is to $D E$, as $C Q$ is to $Q A$ : And $L M$ is alfo to $M D$, as C D to D E: Therefore $L$ M is
to $M \mathrm{D}$, as C Q to $\mathrm{Q} A$ : But L B is to $\mathrm{B} D$, by 5 of Archimedes, before recited, as C D to $\mathrm{D} A$ : It is manifeft therefore, by the precedent Lemma, that $\mathrm{C} D$ is to D , as $\mathrm{L} B$ is to B M: But as C D is to $D \mathrm{Q}$, fo is $C M$ to M P: Therefore $L B$ is to $B M$, as $C M$ to M P:

And it haveing been demonftrated, that C M is to M P, as C E to E A; L B fhall be to B M, as C E to E A. And in like manner it fhall be demonstrated that fo is N O to O F; as alfo the Remainders. And that alfo H K is to K E, as C E to E A, doth plainly appeare by the fame 5. of Archimedes: Which is that that we propounded to be demonftrated.
(a) By 4 . of the
fixth.
(b) By 11 of the
fifth,
(c) By 14 of the
fifth.
By 2. of the fixth

## LEMMA. VI.

And, therefore, let the things ftand as above; and defcribe yet another like Portion, contained betwixt a Right Line, and the Section of the Rightangled Cone D R C, whofe Diameter is R S, that it may cut the Line F G in T; and prolong S R unto the Line C H in V, which meeteth the Section A B C in X , and E F C in Y. I fay, that B M hath to M D, a proportion compounded of the proportion that E A hath to A C; and of that which C D hath to D E.

For, we fhall firft demonftrate, that the Line C H toucheth the Section D R C in the Point C; and that $L$ M is to M D, as alfo N F to FT, and VY to YR, as C D is to ED. And, becaufe now that $L$ B is to $B M$, as C E is to E A; therefore, Compounding and Converting, B M fhall be to L M, as E A to A C: And, as L M is to M D, fo fhall C D be to D E: The proportion, therefore, of $\mathrm{B} M$ to M D , is compounded of the proportion that $B M$ hath to $L M$, and of the proportion that $L M$ hath to $M D$ : Therefore, the proportion of B M to M D, fhall alfo be compounded of the proportion that E A hath to A C, and of that which $C$ D hath to $D E$. In the fame manner it fhal be demonftrated, that $O$ F hath to F T, and alfo X Y to Y R, a proportion compounded of thofe fame proportions; and fo in the reft: Which was to be demonstrated.

By which it appeareth that the Lines fo drawn; which fall betwixt the Sections A B C and D R C, fhall be divided by the Section E F C in the fame Proportion.

And C B is to B D, as fix to fifteen.] For we have fuppofed that B K is
double of K D: Wherefore, by Compofition B D fhall be to K D as three to one; that is, as
fifteen to five: But $\mathrm{B} D$ was to K C as fifteen to four; Therefore $\mathrm{B} D$ is to $\mathrm{D} C$ as fifteen to nine:
And, by Converfion of proportion and Converting, $C B$ is to $B D$, as fix to fifteen.

N

[Figure 53]

And as C B is to B D, fo is

E B to B A; and D Z to D A.]
For the Triangles C B E and D B A being
alike; As C B is to B E, fo fhall D B be to B A:
And, Permutando, as $C B$ is to $B D$, fo fhall
E B be to B A: Againe, as B C is to C E fo
fhall B D be to D A, And, Permutando, as
C B is to B D, fo fhall C E, that is, D Z
equall to it, be to D A .
O
And of D Z and D A, L I and

L A are double.] That the Line LA is
double of $D A$, is manifeft, for that $B D$ is the Diameter of the Portion. And that LI is dovble to D Z fhall be thus demonftrated. For as much as ZD is to D A , as two to five: therefore, Converting and Dividing, A Z, that is, I Z, fhall be to Z D, as three to two:

Again, by dividing, I D fhall be to D Z , as one to two: But Z D was to $\mathrm{D} A$, that is, to D L , as two to five: Therefore, ex equali, and Converting, L D is to D I, as five to one: and, by Converfion of Proportion, D L is to D I, as five to four: But D Z was to D L, as two to five: Therefore, again, ex equali, $D Z$ is to $L I$, as two to four: Therefort $L I$ is double of D Z: Which was to be demonftrated.

P

Q
And, A D is to D I, as five to one.] This we have but juft now demonftrated.

R

For it hath been demonftrated, above, that the Portion whofe
Axis is greater than Sefquialter of the Semi-parameter, if it have
not leffer proportion in Gravity to the Liquid, \&c.] He hath demonstrated this in the fourth Propofition of this Book.

## CONCLVSION II.

If the Portion have leffer proportion in Gravity to the

Liquid, than the Square $S B$ hath to the Square B D, but greater than the Square X O hath to the Square B D, being demitted into the Liquid, fo inclined, as that its Bafe touch not the Liquid, it fhall continue inclined, fo, as that its Bafe fhall not in the leaft touch the Surface of the Liquid, and its Axis fhall make an Angle with the Liquids Surface, greater than the Angle X.

## A

Therfore repeating the firft figure, let the Portion have unto the Liquid in Gravitie a proportion greater than the Square X O hath to the fquare B D, but leffer than the Square made of the Exceffe by which the Axis is greater than Sefquialter of the Semi-

[Figure 54]

Parameter, that is, of S B, hath to
the Square B D: and as the Portion
is to the Liquid in Gravity, fo let
the Square made of the Line $\psi$ be
to the Square B D: $\psi$ fhall be great-
er than X O, but leffer than the
Exceffe by which the Axis is greater than Sefquialter of the Semiparameter, that is, than S B. Let a Right Line M N be applyed to fall between the Conick-Sections
A M Q L and AXD, [parallel to
B D falling betwixt $\mathrm{O} X$ and $B \mathrm{D}$,$] and equall to the Line \psi$ : and let it cut the remaining Conick Section A H I in the point H , and the

Right Line R G in V. It fhall be demonftrated that M H is double to H N, like as it was demonftrated that O G is double to G X.
[Figure 55]

And from the Point M draw M Y touching the Section A M Q L in M; and M C perpendicular to B D: and laftly, having drawn A N \& prolonged it to Q , the Lines $\mathrm{A} N \& N \mathrm{Q}$ fhall be equall to each other. For in regard that in the Like Portions

A M Q L and A XD the Lines A Q and A are drawn from the Bafes unto the Portions, which Lines contain equall Angles with the faid Bafes, Q A fhall have the fame proportion to A M that L A hath to A D: Therefore A N is equall to $N \mathrm{Q}$, and A Q parallel to M Y .

It is to be demonftrated that the Portion being demitted into the Liquid, and fo inclined as that its Bafe touch not the Liquid, it fhall continue inclined fo as that its Bafe fhall not in the leaft touch the Surface of the Liquid, and its Axis fhall make an Angle with the Liquids Surface greater than the Angle X. Let it be demitted into the Liquid, and let it ftand, fo, as that its Bafe do touch the Surface of the Liquid in one Point only; and let the Portion be cut thorow the Axis by a Plane erect unto the Surface of the Liquid,
and Let the Section of the Superficies of the Portion be A P O L, the Section of a Rightangled Cone, and let the Section of the Liquids Surface be A O; And let the Axis of the Portion and Diameter of the Section be B D: and let B D be
cut in the Points K and R as hath been faid; alfo draw P G Parallel to
A O and touching the Section
A P O L in P; and from that Point
draw P T Parallel to B D, and P S perpendicular to the fame B D. Now, forafmuch as the Portion is unto the Liquid in Gravity, as the Square made of the Line $\psi$ is to the Square B D; and fince that as the portion is unto the Liquid in Gravitie, fo is the part thereof fubmerged unto the whole Portion; and that as the part fubmerged is to the whole, fo is the Square T P to the Square B D; It followeth that the Line $\psi$ fhall be equall to T P: And therefore the Lines M N and P T, as alfo the Portions A M Q and A P O fhall likewife be equall to each other. And feeing that in the Equall and Like Portions A P O L and A M Q L the Lines A O and A Q
are drawn from the extremites of their Bafes, fo, as that the Portions cut off do make Equall Angles with their Diameters; as alfo the

Angles at $Y$ and $G$ being equall; therefore the Lines $Y B$ and $G B$, and $B C$ and $B S$ fhall alfo be equall: And therefore $C R$ and $S R$, and $M V$ and $P Z$, and $V N$ and $Z T$, fhall be equall likewife.

Since therefore M V is Leffer than double of $V N$, it is manifeft that $P Z$ is leffer than double of $Z T$. Let $P \omega$ be double of $\omega T$; and drawing a Line from $\omega$ to K, prolong it to E. Now the Centre of Gravity of the whole Portion fhall be the point K ; and the Centre of that part which is in the Liquid fhall be $\omega$, and of that which is above the Liquid fhall be in the Line K E, which let be E: But the Line $\mathrm{K} Z$ fhall be perpendicular unto the Surface of the Liquid: And therefore alfo the Lines drawn thorow the Points E and $\omega$ parall-
lell unto K Z, fhall be perpendicular sunto the fame: Therefore the Portion fhall not abide, but fhall turn about fo, as that its Bafe do not in the leaft touch the Surface of the Liquid; in regard that now when it toucheth in but one Point only, it moveth upwards, on
the part towards A: It is therefore perfpicuous, that the Portion fhall confift fo, as that its Axis fhall make an Angle with the Liquids Surface greater than the Angle X.

B

C

D

E F

G

H

K

L

M

## COMMANDINE.

A
If the Portion have leffer proportion in Gravity to the Liquid, than the Square S B hath to the Square B D, but greater than the Square X O hath to the Square B D.] This is the fecond part of the Tenth propofition; and the other pat is with their Demonftrations, fhall hereafter follow in the fame Order.
$\Psi$ fhall be greater than X O, but leffer than the Excefs by
which the Axis is greater than Sefquialter of the Semi-parameter, that is than S B.] This followeth from the 10 of the fifth Book of Euclids Elements.

B

C

It fhall be demonftrated, that M H is double to H N , like as it was demonftrated, that O G is double to G X.] As in the firft Conclufion of this Propofition, and from what we have but even now written, thereupon appeareth:

D

For in regard that in the like Portions A M Q L and A X D, the Lines A Q and A N are drawn from the Bafes unto the Portions, which Lines contain equall Angles with the faid Bafes, Q A fhall have the fame proportion to A N, that L A hath to A D.]
This we have demonstrated above.

E
Therefore AN is equall to NQ ] For fince that $\mathrm{Q} A$ is to $\mathrm{A} N$, as $L A$ to A D; Dividing and Converting, A N fhall be to N Q as A D to D L: But A D is equall to $\mathrm{D} L$; for that D B is fuppofed to be the Diameter of the Portion: Therefore
alfo (a) A N is equall to N que
(a) By 14 of the fifth.

And A Q parallel to M Y.] By the fifth of the fecond Book of Apollonius his Conicks.
F
And let B D be cut in the Points K and R as hath been faid.]

In the firft Conciufion of this Propofition: And let it be cut in $K$, fo, as that B K be double to $K D$, and in $R$ fo, as that $K R$ may be equall to the Semi-parameter.

G

And, feeing that in the Equall and Like Portions A P O L and

A M Q L, the Lines A O and A Q are drawn from the Extremities
of their Bafes, fo, as that the Portions cut off, do make equall Angles
with their Diameters; as alfo, the Angles at Y and G being equall;
Therefore, the Lines Y B and G B, \& B C \& B S, fhall alfo be equall.]
Let the Line A Q cut the Diameter D B in $\gamma$, and let it cut A O in $\delta$. Now becaufe that in

[Figure 57]
the equall and like Portions A P O L \& A M Q L, from the Extremities of their Bafes, A O and $\mathrm{A} Q$ are drawn, that contain equall Angles with thofe Bafes; and fince the Angles at D, are both Right; Therefore, the Remaining Angles A $\delta \mathrm{D}$ and $\mathrm{A} \gamma \mathrm{D}$ fhall be equall to one another: But the Line P G is parallel unto the Line A O; alfo $M Y$ is parallel to $A$ que and $P S$ and $M C$ to A D: Therefore the Triangles P G S and M Y C, as alfo the Triangles A $\delta \mathrm{D}$ and $\mathrm{A} \gamma \mathrm{D}$, are all alike to each other: (b) And as A D is to A $\delta$,
fo is A D to A $\gamma$ : and, Permutando, the Lines
A D and A D are equall to each other: Therefore,
$\mathrm{A} \delta$ and $\mathrm{A} \gamma$ are alfo equall: But $\mathrm{A} O$ and
A Q are equall to each other; as alfo their halves
A T and A N : Therefore the Remainders $\mathrm{T} \delta$ and $\mathrm{N} \gamma$; that is, TG and MY , are alfo

[Figure 58]
equall. And, as (c) P G is to G S, fo is M Y to Y C: and Permutando, as $P \mathrm{G}$ is to M Y , fo is G S to Y C: And, therefore, G S and Y C are equall; as alfo their halves B S and B C: From whence it followeth, that the Remainders S R and C R are alfo equall: And, confequently, that P Z and $\mathrm{M} V$, and $V \mathrm{~N}$ and $\mathrm{Z} T$, are lkiewife equall to one another.

H
(b) By 4. of the fixth.
(c) By 34 of the
firft,
Since, therefore, that NV is leffer
than double of V N.] For M H is double of H N , and M V is leffer than M H: Therefore, M V is leffer than double of H N , and much leffer than double of V N .

## K

Therefore, the Portion fhall not abide, but fhall turn about,
fo, as that its Bafe do not in the leaft touch the Surface of the Liquid; in regard that now when it toucheth in but one Point only, it moveth upwards on the part towards A.] Tartaglia's his Tranflation hath it thus, Non ergo manet Portio fed inclinabitur ut Bafis ipfius, nec fecundum unum tangat Superficiem Humidi, quon am nunc fecundum unum tacta ipfa reclinatur: Which we have thought fit in this manner to correct, from other Places of Archimedes, that the fenfe might be the more perfpicuous. For in the fixth Propofition of this, he thus writeth (as we alfo have it in the Tranflation,) The Solid A P O L, therefore, fhall turn about, and its Bafe fhall not in the leaft touch the Surface of the Liquid. Again, in the feventh Propofition; From whence it is manifeft, that its Bafe fhall turn about in fuch manner, a that its Bafe doth in no wife touch the Surface of the Liquid; For that now when it toucheth but in one Point only, it moveth downwards on the part towards L. And that the Portion moveth upwards, on the part towards A, doth plainly appear: For fince that the Perpendiculars unto the Surface of the Liquid, that pafs thorow $\omega$, de fall on the part towards A , and thofe that pafs thorow E, on the part towards L; it is neceffary that the Centre $\omega$ do move upwards, and the Centre E downwards.

## L

It is therefore perfpicuous, that the Portion fhall confift, fo, as that its Axis fhall make an Angle with the Liquids Surface greater than the Angle X.] For drewing a Line from A to X, prolong it untill it do cut the Diamter
[Figure 59]

B D in $\lambda$; and from the Point O , and parallel to A $\lambda$, draw O X; and let it touch the Section in O, as in the first Figure: And the (d) Angle at X,
fhall be equall alfo to the angle $\lambda$ : But the angle at Y is equall to the Angle at $\gamma$; and the (e) Angle

A Г D greater than the Angle A $\lambda$ D, which falleth without it: Therefore the Angle at Y fhall be greater than that at X . And becaufe now the Portion turneth about, fo, as that the Bafe doth not touch the Liquid, the Axis fhall make an Angle with its Surface greater than the Angle G; that is, than the Angle Y: And, for that reafon, much greater than the Angle X.
(d) By 29 of the firft.
(e) By 16. of the firf.

## CONCLUSION III.

If the Portion have the fame proportion in Gravity to the Liquid, that the Square X O hath to the Square BD , being demitted into the Liquid, fo inclined, as that its Bafe touch not the Liquid, it fhall ftand and continue inclined, fo, as that its Bafe touch the Surface of the Liquid, in one Point only, and its Axis fhall make an Angle with the Liquids Surface equall to the Angle X. And, if the Portion have the fame proportion in Gravity to the Liquid, that the Square P F hath to the Square B D, being demitted into the Liquid, \& fet fo inclined, as that its Bafe touch not the Liquid, it fhall ftand inclined, fo, as that its Bafe touch the Surface of the Liquid in one Point only, \& its Axis fhall make an Angle with it, equall to the Angle $\Phi$.

Let the Portion have the fame proportion in Gravity to tho Liquid that the Square XO hath to the Square B D; and let it be demitted into the Liquid fo inclined, as that its Bafe touch
[Figure 60]
not the Liquid. And cutting it by a Plane thorow the Axis, erect unto the Surface of the Liquid, let the Section of the Solid, be the Section of a Right-angled Cone, A P M L; let the Section of the Surface of the Liquid be I M; and the Axis of the Portion and Diameter of the Section B D; and let B D be divided as before; and draw PN parallel to IM
and touching the Section in P, and T P parallel to B D; and P S perpendicular unto B D. It is to be demonftrated that the Portion fhall
[Figure 61]
not ftand fo, but fhall encline until that the Bafe touch the Surface of the Liquid, in one Point only, for let the fuperior figure ftand as it was, and draw O C, Perpendicular to B D; and drawing a Line from $A$ to $X$, prolong it to Q : A X fhalbe equall to X que Then draw O X parallel to A que And becaufe the Portion is fuppofed to have the fame proportion in Gravity to the Liquid that the fquare X O hath to the Square B D; the part thereof fubmerged fhall alfo have the fame proportion to the whole; that is, the Square T P to the Square

B D; and fo T P fhall be equal to X O: And fince that of the Portions I P M and A O Q the Diameters are equall, the portions fhall alfo be
equall. Again, becaufe that in the Equall and Like Portions A O Q L
and AP ML the Lines A Q and I M, which cut off equall Portions, are drawn, that, from the Extremity of the Bafe, and this not from the Extremity; it appeareth that that which is drawn from the end or Extremity of the Bafe, fhall make the Acute Angle with the Diameter of the whole Portion lefset. And the Angle at X
being leffe than the Angle at N, B C fhall be greater than B S; and C R leffer than S R: And, therfore O G fhall be leffer than P Z; and GX greater than ZT : Therfore PZ is greater than double of Z T; being that O G is double of G X. Let P H be double to H T; and drawing a Line from H to K , prolong it to $\omega$. The Center of Gravity of the whole Portion fhall be K; the Center of the part which is within the Liquid H , and that of the part which is above the Liquid in the Line $\mathrm{K} \omega$; which fuppofed to be $\omega$. Therefore it fhall be demonftrated, both, that K H is perpendicular to the Surface of the Liquid, and thofe Lines alfo that are drawn thorow the Points Hand $\omega$ parallel to K H: And therfore the Portion fhall not reft, but fhall encline untill that its Bafe do touch the Surface of the Liquid
in one Point; and fo it fhall continue. For in the Equall Portions A O QL and A PML, the

[Figure 62]

Lines A Q and A M, that cut off equall Portions, fhall be dawn from the Ends or Terms of the Bafes; and A O Q and A P M fhall be demonftrated, as in the former, to
be equall: Therfore A Q and A M, do make equall Acute Angles with the Diameters of the Portions; and
the Angles at X and N are equall. And, therefore, if drawing HK , it be prolonged to $\omega$, the Centre of Gravity of the whole Portion fhall be K ; of the part which is within the Liquid H ; and of the part which is above the Liquid in $\mathrm{K} \dot{\omega}$ as fuppofe in $\omega$; and H K perpendicular to

[Figure 63]
the Surface of the Liquid. Therfore
along the fame Right Lines fhall the
part which is within the Liquid move upwards, and the part above it downwards: And therfore the Portion fhall reft with one of its Points touching the Surface of the Liquid, and its Axis fhall make with the
fame an Angle equall to X . It is to be demonftrated in the fame manner that the Portion that hath the fame proportion in Gravity to the Liquid, that the Square P F hath to the Square B D, being demitted into the Liquid, f , as that its Bafe touch not the Liquid, it fhall ftand inclined, fo, as that its Bafe touch the Surface of the Liquid in one Point only; and its Axis fhall make therwith an Angle equall to the Angle $\varphi$.

A

B

C

D

E

F

## COMMANDINE.

A

That is the Square T P to the Square B D.] By the twenty fixth of the Book
fhall be equall to the Square X O: And for that reafon, the Line T P equall to the Line X O.
(a) By 9 of the
fifth.

B

The Portions fhall alfo be equall.] By the twenty fifth of the fame Book.

## C

Again, becaufe that in the Equall and Like Portions, A O Q L and A P M L.] For, in the Portion A P M L, defcribe the Portion A O Q equall to the Portion I P M: The Point Q falleth beneath M; for otherwife, the Whole would be equall to the Part. Then draw I V parallel to A Q, and cutting the Diameter is $\psi$; and let I M cut the fame s; and A Q in s. I fay
that the Angle A $\cup$ D, is leffer than the Angle

[Figure 64]

I $\sigma$ D. For the Angle $\mathrm{I} \psi \mathrm{D}$ is equall to the
Angle A $\sim$ D: (b) But the interiour Angle
$\mathrm{I} \psi \mathrm{D}$ is leffer than the exteriour $\mathrm{I} \sigma \mathrm{D}$ : There-
fore, (c) A $\cup \mathrm{D}$ fhall alfo be lefter than $\mathrm{I} \sigma \mathrm{D}$.
(b) By 29 of the firft.
(c) By 16 of the
firf.

D
And the Angle at X , being leffe
than the Angle at N.] Thorow O draw twe
Lines, O C perpendicular to the Diameter B D, and
OX touching the Section in the Point O , and cutting
the Diameter in X: (d) O X fhall be parallel
to A que and the (e) Angle at X , fhall be equall to
that at $v$ : Therefore, the (f) Angle at X ,
fhall be leffer than the Angle at $\varsigma$; that is, to
that at N : And, confequently, X fhall fall beneath N : Therefore, the Line $\mathrm{X} B$ is greater than $N B$. And, fince $B C$ is equall to $X B$, and $B S$ equall to $N B ; B C$ fhall be greater than $B S$.
(d) By 5 of our fecond of Conicks.
(e) By 29 of the
firf.
(f) By 39 of our
firft of Conicks.
Therefore, A Q and A M do make equall Acute Angles with
the Diameters of the Portions.] We demonftrate this as in the Commentaries upon the fecond Conclufion.

## E

It is to be demonftrated in the fame manner, that the Portion
that hath the fame proportion in Gravity to the Liquid, that the Square P F hath to the Square B D, being demitted into the Liquid, fo,

[Figure 65]
as that its Bafe touch not the Liquid, it fhall ftand inclined, fo, as that its Bafe touch the Surface of the Liquid in one point only; and its Axis fhall make therewith an angle equall to the Angle $\varphi$.] Let the Portion be to the Liquid in Gravity, as the Square P F to the Square B D: and being demitted into the Liquid, fo inclined, as that its Bafe touch not the Liquid, let it be cut thorow the Axis by a Plane erect to the Surface of the Liquid, that that the Section may be A M O L, the Section of a Rightangled Cone; and, let the Section of the Liquids Surface be I O; and the Axit of the Portion and Diameter of the Section B D; which let be cut into the fame parts as we faid before, and draw M N parallel to I O, that it may touch the Section in the Point M ; and M T parallel to B D, and P M S perpe ndicular to the fame. It is to be demonstrated, that the Portion fhall not reft, but fhall incline, fo, as that it touch the Liquids Surface, in one Point of its Bafe only. For,
[Figure 66]
draw P C perpendicular to B D ; and drawing a Line from A to F, prolong it till it meet with the Section in que and thorow P draw $\mathrm{P} \varphi$ parallel to A Q: Now, by the things allready demonftrated by us, A F and F Q fhall be equall to one another. And being that the Portion hath the fame proportion in Gravity unto the Liquid, that the Square P F hath to the Square B D; and feeing that the part fubmerged, hath the fame pro-
partion to the whole Portion; that is, the Squàre
M T to the Square B D; (g) the Square M T
fhall be equall to the Square P F; and, by the
fame reafon, the Line M T equall to the Line
P F. So that there being drawn in the equall \& like
portions A P Q Land A M O L, the Lines A Q and I O which cut off equall Portions, the firft from the Extreme term of the Bafe, the laft not from the Extremity; it followeth, that A Q drawn from the Extremity, containeth a leffer Acute Angle with the Diameter of the Portion, than I O: But the Line P $\varphi$ is parallel to the Line A Q , and M N to I O: Therefore, the Angle at $\varphi$ fhall be leffer than the Angle at N ; but the Line B C greater than B S; and SR, that is, M X, greater than CR, that is, than PY: and, by the fame reafon, X T leffer than Y F. And, fince P Y is double to Y F, M X fhall be greater than double to Y F, and much greater than double of X T. Let M H be double to H T, and draw a Line from H to K, prolonging it. Now, the Centre of Gravity of the whole Portion fhall be the Point K ; of the part within the Liquid H ; and of the Remaining part above the Liquid in the Line H K produced, as fuppofe in $\omega$ It fhall be demonftrated in the fame manner, as before, that both the Line K H and thofe that are drawn thorow the Points H and $\omega$ parallel to the faid K H , are perpendicular to the Surface of the Liquid: The Portion therefore, fhall not reft; but when it fhall be enclined fo far as to touch the Surface of the Liquid in one Point and no more, then it fhall ftay. For the Angle at N
[Figure 67]
fhall be equall to the Angle at $\varphi$; and the Line B S equall to the Line B C; and S R to C R: Wherefore, M H fhall be likewife equall to $\mathrm{P} Y$. Therefore, having drawn HK and prolonged it; the Centre of Gravity of the whole Portion fhall be K ; of that which is in the Liquid H ; and of that which is above it, the Centre fhall be in the Line prolonged: let it be in $\omega$. Therefore, along that fame Line K H, which is perpendicular to the Surface of the Liquid, fhall the part which is within the Liquid move upwards, and that which is above the Liquld downwards: And, for this caufe, the Portion, fhall be no longer moved, but fhall ftay, and reft, fo, as that its Bafe do touch the Liquids Surface in but one Point; and its Axis maketh an Angle therewith equall to the Angle $\varphi$; And, this is that which we were to demonftrate.

F
(g) By 9 of $t$
fifth.

## CONCLVSION IV.

If the Portion have greater proportion in Gravity to the Liquid, than the Square F P to the Square B D, but leffer than that of the Square X O to the Square B D, being demitted into the Liquid, and inclined, fo, as that its Bafe touch not the Liquid, it fhall ftand and reft, fo , as that its Bafe fhall be more fubmerged in the Liquid.

Again, let the Portion have greater proportion in Gravity to the Liquid, than the Square F P to the Square B D, but leffer than that of the Square X O to the Square B D; and as the Portion is in Gravity to the Liquid, fo let the Square made of the Line $\psi$ be to the Square B D. $\Psi$ fhall be greater than F P, and leffer than X O. Apply, therefore, the right Line I V to fall betwixt the Portions A V Q L and A X D; and let it be equall to $\psi$, and parallel to B D; and let it meet the Remaining Section in Y: V Y fhall alfo be proved double
to Y I, like as it hath been demonftrated, that O G is double off G X. And, draw from $V$, the Line $V \omega$, touching the Section A V Q L in V; and drawing a Line from A to I, prolong it unto que We prove in the fame manner, that the Line $\mathrm{A} I$ is equall to I que and that A Q is parallel to $\mathrm{V} \omega$. It is to be demonftrated, that the Portion being demitted into the Liquid, and fo inclined, as that its Bafe touch not the Liquid, fhall ftand, fo, that its Bafe fhall be more fubmerged in the Liquid, than to touch it Surface in
but one Point only. For let it be de-

[Figure 68]
mitted into the Liquid, as hath been faid; and let it firft be fo inclined, as that its Bafe do not in the leaft touch the Surface of the Liquid. And then it being cut thorow the Axis, by a Plane erect unto the Surface of the Liquid, let the Section of the Portion be A N Z G; that of the Liquids Surface E Z; the Axis of the Portion and Diameter of the Section B D; and let B D be cut in the Points K and R, as before; and draw N L parallel to E Z, and touching the Section A N Z G in N , and $\mathrm{N} S$ perpendicular to

[Figure 69]

B D. Now, feeing that the Portion is in Gravity unto the Liquid, as the Square made of the Line is to the Square B D; $\psi$ fhall be equall to N T : Which is to be demonftrated as above: And, therefore, NT is alfo equall to V I: The Portions, therefore, $A V Q$ and $E N Z$ are equall to one another. And, fince that in the Equall and like Portions A V
$Q L$ and $A N Z G$, there are drawn $A Q$ and $E Z$, cutting off equall Portions, that from the

[Figure 70]

Extremity of the Bafe, this not
from the Extreme, that which is drawn from the Extremity of the Bafe, fhall make the Acute Angle with the Diameter of the Portion leffer: and in the Triangles N L S and $V \omega \mathrm{C}$, the Angle at L is greater than the Angle at $\omega$ : Therefore, B S fhall be leffer than B C; and S R leffer than C R: and, confequently, NX greater than V H; and X T leffer than H I. Seeing, therefore, that V Y is double to Y I; It is manifeft, that N X is greater than double to X T. Let N M be double to M T: It is manifeft, from what hath been faid, that the Portion fhall not reft, but will incline, untill that its Bafe do touch the Surface of the Liquid: and it toucheth it in one Point only, as appeareth in the Figure: And other things
[Figure 71]
ftanding as before, we will again demonftrate, that N T is equall to V I; and that the Portions A V Q and AN Z are equall to each other. Therefore, in regard, that in the Equall and Like Portions A V Q L and A N Z G, there are drawn $A Q$ and $A Z$ cutting off equall Portions, they fhall with the Diameters of the Portions, contain equall
Angles. Therefore, in the Triangles
NLS and $V \omega C$, the Angles at
the Points L and $\omega$ are equall; and the Right Line B S equall to B C; S R to C R; N X to V H; and X T to H I: And, fince V Y is double to Y I, N X fhall be greater than double of X T. Let therefore, $\mathrm{N} M$ be double to M T. It is hence again manifeft, that the Portion will not remain, but fhall incline on the part towards A: But it was fuppofed, that the faid Portion did touch the Surface of the Liquid in one fole Point: Therefore, its Bafe muft of neceffity fubmerge farther into the Liquid.

## CONCLVSION V.

If the Portion have leffer proportion in Gravity to the Liquid, than the Square F P to the Square B D , being demitted into the Liquid, and inclined, fo, as that its Bafe touch not the Liquid, it fhall ftand fo inclined, as that its Axis fhall make an Angle with the Surface of the Liquid, leffe than the Angle $\psi$; And its Bafe fhall not in the leaft touch the Liquids Surface.

Finally, let the Portion have leffer proportion to the Liquid in Gravity, than the Square F P hath to the Square B D; and as the Portion is in Gravity to the Liquid, fo let the Square made of the Line $\psi$ be to the Square B D. $\psi$ fhall be leffer than P F. Again, apply any Right Line as G I, falling betwixt the Sections A G QL and A X D, and parallel to B D; and let it cut the Middle Conick Section in the Point H, and
the Right Line R Y in Y. We

[Figure 72]
fhall demonftrate GH to be double to HI , as it hathbeen demonftrated, that O G is double to G X. Then draw $\mathrm{G} \omega$ touching the Section A G Q L in G ; and G C perpen dicular to B D; and drawing a Line from A to I, prolong it to que Now
A I fhall be equall to I que and A $Q$ parallel to $G \omega$. It is to be demonftrated, that the Portion being demitted into the Liquid, and inclined, fo, as that its Bafe touch the Liquid, it fhall ftand fo incli-

[Figure 73]
ned, as that its Axis fhall make an Angle with the Surface of the Liquid leffe than the Angle $\varphi$; and its Bafe fhall not in the leaft touch the Liquids Surface. For let it be demitted into the Liquid, and let it ftand, fo, as that its Bafe do touch the Surface of the Liquid in one Point only: and the Portion being cut thorow the Axis by a Plane erect unto the Surface of the Liquid, let the Section of

[Figure 74]
the Portion be A N Z L, the Section
of a Rightangled Cone; that of
the Surface of the Liquid A Z; and
the Axis of the Portion and Dia-
meter of the Section B D; and let
B D be cut in the Points K and R as hath been faid above; and draw N F parallel to A Z, and touching the Section of the Cone in the Point N ; and N T parallel to B D; and $\mathrm{N} S$ perpendicular to the fame. Becaufe, now, that the Portion is in Gravity to the Liquid, as the Square made of $\psi$ is to the Square B D; and fince that as the Portion is to the Liquid in Gravity, fo is the Square N T to the Square B D, by the things that have been faid; it is plain, that N T is equall to the Line $\psi$ : And, therefore, alfo, the Portions A NZ and $\mathrm{A} \mathrm{G} \mathrm{Q} \mathrm{are} \mathrm{equall}. \mathrm{And}$, Like Portions A G Q L and A N Z L; there are drawn from the Extremities of their Bafes, A Q and A Z which cut off equall Portions: It is obvious, that with the Diameters of the Portions they
make equall Angles; and that in the Triangles NFS and G $\omega$ C the Angles at F and $\omega$ are equall; as alfo, that S B and B C, and $S R$ and $C R$ are equall to one another: And, therefore, $N X$ and G Y are alfo equall; and X T and Y I. And fince G H is double to H I, N X fhall be leffer than double of X T. Let N M therefore be double to $\mathrm{M} T$; and drawing a Line from M to K , prolong it unto E. Now the Centre of Gravity of the whole fhall be the Point K; of the part which is in the Liquid the Point M; and that of the part which is above the Liquid in the Line prolonged as fuppofe in E. Therefore, by what was even now demonftrated it is manifeft that the Portion fhall not ftay thus, but fhall incline, fo as that its Bafe do in no wife touch the Surface of the Liquid And that the Portion will ftand, fo, as to make an Angle with the Surface of the Liquid leffer than

[Figure 75]
the Angle $\varphi$, fhall thus be demon ftrated. Let it, if poffible, ftand, fo, as that it do not make an Angle leffer than the Angle $\varphi$; and difpofe all things elfe in the fame manner a before; as is done in the prefet
Figure. We are to demonftrat in the fame method, that NT is equall to $\psi$; and by the fame reafor equall alfo to G I. And fince that in the Triangles $P \varphi C$ and NFS, the Angle F is not leffer than the Angle $\varphi$, B F fhall not be greater than B C: And, therefore, neither fhall S R be leffer than C R; nor N X than P Y: But fince P F is greater than N T, let P F be Sefquialter of P Y: N T fhall be leffer than Sefquialter of NX : And, therefore, NX fhall be greate than double of XT . Let N M be double of M T ; and drawing Line from $M$ to $K$ prolong it. It is manifeft, now, by what hath been faid, that the Portion fhall not continue in this pofition, but fhall turn about, fo, as that its Axis do make an Angle with the Surface of the Liquid, leffer than the Angle $\varphi$.

